DOCUMENT RESUME

ED 087 631 SE 017 302

AUTHOR	Schaaf, William L.
TITLE	A Bibliography of Recreational Mathematics, Volume 3.
INSTITUTION	National Council of Teachers of Mathematics, Inc., Washington, D.C.
PUB DATE	73
NOTE	187p.
AVAILABLE FROM	National Council of Teachers of Mathematics, 1906 Association Drive, Reston, Virginia 22091
EDRS PRICE	MF-$0.65 HC Not Available from EDRS.
DESCRIPTORS	*Annotated Bibliographies; Classroom Games; *Games; Geometric Concepts; Instructional Materials; *Literature Guides; Literature Reviews; *Mathematical Enrichment; *Mathematics Education; Number Concepts; Probability; Reference Books

ABSTRACT

This book is a partially annotated bibliography of books, articles and periodicals concerned with mathematical games, puzzles, tricks, amusements, and paradoxes. Because the literature in recreational mathematics has proliferated to amazing proportions since Volume 2 of this series (ED 040 874), Volume 3 is more than just an updating of the earlier monographs. The overall organization of the book has been retained, although there has been a notable rearrangement of subtopics in the interest of economy and clarity. Major changes include (1) the addition of two new sections on classroom games and recreational activities which will be quite useful for teachers; (2) a chronological synopsis of Martin Gardner's popular column in Scientific American; and (3) a glossary of terms related to recreational mathematics. The main topic headings are: Arithmetic Recreations; Number Theory as Recreation; Geometric Recreations; Topological Recreations; Magic Squares and Related Configurations; Pythagorean Recreations; Recreations in Antiquity; Combinatorial Recreations; Manipulative Recreations; and Mathematics in Related Fields. (JP)

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A BIBLIOGRAPHY OF recreational mathematics

VOLUME

3

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Library of Congress Cataloging in Publication Data (Revised):

Schaaf, William Leonard, 1898–
A bibliography of recreational mathematics.

Vols. 2 and 3 lack edition statement.
First-3d editions published under title: Recreational mathematics.
1. Mathematical recreations—Bibliography.
I. Title.
Z6654.M3S32 016.7937'4 73-12168

Printed in the United States of America

Preface

Since the appearance of volume 2 in 1970, general interest in mathematical recreations has not abated. On the contrary, not only has the literature become more sophisticated, it has proliferated to amazing proportions. As Maxey Brooke suggests, it is like Alice: You have to keep running as hard as you can to stay in the same place. Thus the present volume is more than just an updating of the earlier monographs.

For the reader's convenience, the overall organization of the book has been retained, although there has been a notable rearrangement of subtopics in the interest of economy and greater clarity. Major changes include (1) the addition of two new sections on classroom games and recreational activities, which will doubtless appeal to teachers; (2) a chronological synopsis of Martin Gardner's popular column in *Scientific American*; and (3) a glossary of terms related to recreational mathematics.

As heretofore, some references have been listed under two different headings. Also, several hundred entries that were somehow previously overlooked have been belatedly included. Finally, a few items listed in the earlier volumes have been deliberately repeated here to enable the reader to retrieve related information concerning a given topic.

To acknowledge my indebtedness to many friends and correspondents would entail a lengthy list of names, and so I hereby express my gratitude to them all. My thanks are especially due to Maxey Brooke, Mannis Charosh, Martin Gardner, James A. H. Hunter, and Charles W. Trigg, whose help and suggestions have been invaluable. My thanks are also extended to the National Council of Teachers of Mathematics for its customary cooperation, and to my wife for her unflagging encouragement and patience.

William L. Schaaf

Boca Raton, Florida
April 1973

Contents

Problems

Problems worthy
of attack
prove their worth
by hitting back.

PIET HEIN
Grooks 1, p. 2 (1966)

Last Things First

Solutions to problems
are easy to find:
the problem's a great
contribution.
What is truly an art
is to wring from your mind
a problem to fit
a solution.

PIET HEIN
Grooks 3, p. 15 (1970)

Principal Abbreviations Used

A.M.M. = *American Mathematical Monthly*
A.T. = *Arithmetic Teacher*
Fib.Q. = *Fibonacci Quarterly*
J.R.M. = *Journal of Recreational Mathematics*
M.Gaz. = *Mathematical Gazette*
M.Mag. = *Mathematics Magazine*
M.S.J. = *Mathematics Student Journal*
M.T. = *Mathematics Teacher*
M.Tchg. = *Mathematics Teaching* (England)
NCTM = *National Council of Teachers of Mathematics*
N.M.M. = *National Mathematics Magazine*
P.M.E.J. = *Pi Mu Epsilon Journal*
R.M.M. = *Recreational Mathematics Magazine*
Sci.Am. = *Scientific American*
Sci.Mo. = *Scientific Monthly*
S.S.M. = *School Science and Mathematics*
Scrip.M. = *Scripta Mathematica*

Chapter 1

Arithmetical Recreations

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Magic Squares and Related Configurations

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Chapter 11

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Chapter 12

Recreations in the Classroom

12.1 Elementary School Activities

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Ackerman, Judy. Computers teach math. *A.T.* 15:467–68; May 1968.
A game called hexapawn; construction of a game-learning computer of the ticktacktoe type.

Aichele, Douglas B. "Pica-Centro"—a game of logic. *A.T.* 19:359–62; May 1972.

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Cameron, A. J. *Mathematical Enterprises for Schools.* New York: Pergamon Press, 1966. 188 pp.
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Johnson, Donovan. *Games for Learning Mathematics.* Portland, Maine: J. Weston Walch, 1963. 176 pp.

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Liedtke, Werner. What can you do with a geoboard? *A.T.* 16:491–93; Oct. 1969.

Manheimer, Wallace. Club project in a modern use of mathematics. *M.T.* 50:350–55; May 1957.

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Seymour, Dale. *Finite Differences*. Palo Alto, Calif.: Creative Publications, 1973.
Problem-solving activities; grades 7–12.

Sinkhorn, Richard, and Cecil B. Read. Mathematical bingo. *S.S.M.* 55:650–52; Nov. 1955.

Smith, Eugene P. "Some Puzzlers for Thinkers." In *Enrichment Mathematics for the Grades*, pp. 211–20. Twenty-seventh Yearbook. Washington, D.C.: NCTM, 1963.
For the junior high school level; about two dozen assorted problems, including magic squares.

Steiger, Sister Anne Agnes von. Christmas puzzle. *M.T.* 60:848–49; Dec. 1967.

Steinen, Ramon F. More about 1965 and 1966. *M.T.* 59:737–38; Dec. 1966.

Stokes, William T. *Notable Numbers*. Palo Alto, Calif.: Creative Publications, 1973.
Number relations, patterns, curiosities, and so on; grades 5–12.

Trigg, Charles W. Holiday greetings from thirty scrambled mathematicians. *S.S.M.* 54:679; Dec. 1954.

———. Triangular arrangements of numbered disks. *M.T.* 65:157–60; Feb. 1972.

Wessel, G. Base minus-ten numeration system. *S.S.M.* 68:701–6; Nov. 1968.

Winick, David F. "Arithmecode" puzzle. *A.T.* 15:178–79; Feb. 1968.
Similar to a cross-number puzzle.

12.3 Mathematics Clubs, Plays, Programs, Projects

Bleustein, Robert. The King and *i*; a play in three scenes. *M.S.J.* vol. 17, no. 4, pp. 3–4; May 1970.

Bruyr, Donald. *Geometrical Models and Demonstrations*. Portland, Maine: J. Weston Walch, 1964. 173 pp.
Curves, surfaces, solids, instruments, and so on; over 150 diagrams.

Cordell, Christobal. *Dramatizing Mathematics*. Portland, Maine: J. Weston Walch, 1963. 170 pp.

A collection of 17 skits, contests, and so on, appropriate for mathematics club programs and school assemblies.

Dienes, Z. P., and E. W. Golding. *Sets, Numbers and Powers*. New York: Herder & Herder, 1966. 122 pp. (Paper)

Practical suggestions for lessons and games to help develop the ideas embodied in the title; companion volume to a handbook.

Granito, Dolores. What to do in a mathematics club. *M.T.* 57:35–40; Jan. 1964.

Humphrey, J. H., and Dorothy Sullivan. *Teaching Slow Learners through Active Games*. Springfield, Ill.: Charles C. Thomas (301 E. Lawrence Ave.), 1970. 184 pp.

Describes over 100 games related to reading, science, and mathematics.

Johnson, Donovan, C. H. Lund, and W. D. Hamerston. *Bulletin Board Displays for Mathematics*. Belmont, Calif.: Dickenson Publishing Co., 1967. 99 pp.

Kapur, J. N. *Suggested Experiments in School Mathematics*. 2 vols. Karol Bagh, New Delhi: Arya Book Depot, 1969. 144 + 232 pp.

Experiments. grouped by topics, to facilitate the understanding of modern mathematical concepts.

Schaaf, William L. Mathematical plays and programs. *M.T.* 44:526–28; Nov. 1951.

Contains an annotated list of 50 plays, pageants, and skits and a list of 20 references on programs for assemblies and mathematics clubs.

Todd, Audrey. *The Maths Club*. London: H. Hamilton, 1968.

Willerding, Margaret. Dramatizing mathematics. *S.S.M.* 60:99–104; Feb. 1960.

An annotated list of 77 plays, pageants, and skits and a bibliography of 7 references on quiz shows and assembly programs.

12.4 Mathematics Contests, Competitions, Leagues

Altendorf, J. J.. and M. A. McCormick. Stimulating enthusiasm about math.; Missouri Southern College math league. *School and Community* [Missouri State Teachers Association] 55:26; Apr. 1969.

Burkhill, J. C., and H. M. Cundy. *Mathematical Scholarship Problems*. New York: Cambridge University Press, 1961. 118 pp.

Cash Prizes to be Awarded to Florida Students in the High School Mathematics Contest. *Florida Council of Teachers of Mathematics Newsletter*, vol. 14, no. 2, pp. 11–13; Winter 1972.

Charosh, Mannis, ed. *Mathematical Challenges*. Washington, D.C.: NCTM, 1965. 135 pp. (Paper)

Collection of problems appropriate for grades 7 through 12.

Cromack, Norman E. An assessment of a mathematics league as judged by its participants. *M.T.* 63:432–38; May 1970.

———. Mathematics leagues in New Jersey. *New Jersey Mathematics Teacher* 24:21–23; May 1967.

Hlavaty, Julius H. The Czechoslovak national mathematical olympiads. *M.T.* 61:80–85; Jan. 1968.

McCormick, Martha. Students become math-minded through league influence. *M.T.* 64:245–46; Mar. 1971.

Maths Olympiad a True Test. *Times* (London) *Education Supplement* 2739:1136; 17 November 1967.

Paarlberg, Teunis. The mathematics league. *M.T.* 60:38–40; Jan. 1967.

Turner, Nura D. The U.S.A. mathematical olympiad. *A.M.M.* 79:301–2; Mar. 1972.

———. Why can't we have a U.S.A. mathematical olympiad? *A.M.M.* 78:192–95; 1971.

APPENDIX A

Contemporary Works on Mathematical Recreations

Books and monographs devoted exclusively to a specific topic (e.g., *Dissections* or *Magic Squares* or *Tangrams*) are listed under the appropriate chapter and subtopic headings.

Barr, George. *Entertaining with Number Tricks.* New York: McGraw-Hill Book Co., 1971. 143 pp.

Basile, Joseph. *100 (Cent) problèmes de mathématiques amusantes.* Paris: L'Inter, 1967.

Beard, Robert S. *Patterns in Space.* San Jose, Calif.: Fibonacci Association, 1971. 200 pp. (approx.)
An unusual collection of geometric drawings, patterns, curves, solids, and so on.

Beck, Anatole, Michael Bleicher, and Donald Crowe. *Excursions into Mathematics.* New York: Worth Publishers, 1969. 489 pp.
Chapters on polyhedra, perfect numbers, area concept, a variety of geometries, mathematical games, and numeration systems.

Beer, Fritz [Von Complexus]. *Fröhliches Kopfzerbrechen; 100 Aufgaben für scharfe Denker, mit einem Anhang: Lösungen und Erläuterungen.* Vienna and Leipzig: M. Perles, 1934.

Bold, Benjamin. *Famous Problems of Mathematics.* New York: Van Nostrand Reinhold, 1969. 112 pp.

British Broadcasting Corporation (BBC), School Broadcasting Department. *Mathematics Miscellany: A Source Book for Teachers, By Members of the School Broadcasting Dept., BBC Television.* London: BBC, 1966.

Charosh, Mannis. *Mathematical Games for One or Two.* New York: Thomas Y. Crowell Co., 1972. 33 pp.

Conference for the Development of Mathematical Puzzles, Problems and Games: A Report. Stanford, Calif.: Stanford University, 1965.

Dobrovolný, Bohumil. *Matematické rekreace; zajímavé problémy s 90 obrázky a s řešením.* Prague: Práce, 1961.

Emmet, E. R. *Puzzles for Pleasure.* New York: Emerson Books, 1972. 310 pp.

Férez, Antonio H. *Maravillas recreativas del calculo aritmético; . . . con nuevas curiosidades.* Madrid: Editorial E. C. M., 1952.

Friedland, Aaron J. *Puzzles in Math and Logic.* New York: Dover Publications, 1970. 72 pp. (Paper)

One hundred original problems; not a reprint book.

Gardner, Martin. *Martin Gardner's Sixth Book of Mathematical Games from Scientific American.* San Francisco: W. H. Freeman & Co., 1971. 262 pp.

Two dozen recreations never before published in book form.

Gibson, Walter B. *Fell's Guide to Papercraft, Tricks, Games and Puzzles.* New York: Frederick Fells, 1963. 125 pp.

Haber, Heinz. *Das mathematische Kabinett.* Stuttgart: Deutsche Verlagsanstalt, 1967.

Hochkeppel, W. *Denken als Spiel.* Ebenhausen, 1970. 224 pp.

Psychological, logical, and physical problems; linguistic puzzles.

Hollis, Martin. *Tantalizers: A Book of Original Logical Puzzles.* London: George Allen & Unwin, 1970. 153 pp.

Hunter, J. A. H. *Figures Are Fun: Books 1–5.* With teacher's manual. Toronto: Copp Clark Publishing Co., 1959.

Designed for use in grades 4–9.

———. *Figures for Fun.* London: J. M. Dent & Sons, 1957.

Mathematical puzzles couched in little stories; for young readers.

Hurley, James F., ed. *Litton's Problematical Recreations.* New York: Van Nostrand Reinhold, 1971. 337 pp.

Over 250 puzzles, many of which have appeared previously in *Mathematical Bafflers,* by Angela Dunn (McGraw-Hill Book Co., 1964).

Kabinett, D. *Mathematische Auslese mathematischer und Denksport-Aufgaben.* 2 vols. Stuttgart, 1967. 128 + 110 pp.

Kordemsky, Boris A. *Matematicheskaya smekalka.* Moscow, 1957. French translation: *Sur le sentier des mathématiques.* 2 vols. Paris: Dunod, 1963.

———. *The Moscow Puzzles; 359 Mathematical Recreations.* New York: Charles Scribner's Sons, 1972. 309 pp.

A very popular puzzle book in the USSR; in English.

Krbek, F. v. U. *Zahlen und Überzahlen.* Leipzig, 1964. 146 pp.

Krulik, Stephen. *A Handbook of Aids for Teaching Junior-Senior High School Mathematics.* Philadelphia: W. B. Saunders Co., 1971. 120 pp.

A collection of some 40 interesting games and devices, including the Möbius strip, the Tower of Hanoi, Chinese tangrams, curve stitching, and the number-base calendar.

Lamb. Sydney H. *The Magic of Numbers.* New York: Arco Publishing Co., 1965. 71 pp.

General introductory material for very young readers.

Linn, Charles F. *Odd Angles: Thirty-three Mathematical Entertainments.* Garden City, N.Y.: Doubleday & Co., 1971. 127 pp.

Litton's Problematical Recreations. Edited by J. F. Hurley. New York: Van Nostrand Reinhold, 1971. 337 pp.

Longley-Cook, L. H. *New Math Puzzle Book.* New York: Van Nostrand Reinhold, 1970. 176 pp.

Recreations combined with ideas from "new math."

Lukács, Clara, and Emma Tarján. *Mathematical Games.* Translated by John Dobai. New York: Walker & Co., 1969. 200 pp.

———. *Spiele mit Zahlen* (Mathematische Spiele, wie Karten—und Rechentricks, Denksportaufgaben, u.s.w.). Benziger-Tabu, 1968. 168 pp.

Mira, Julio A. *Mathematical Teasers.* New York: Barnes & Noble, 1970. 279 pp. (Paper)

Problems, puzzles, and tricks; with explanations.

Morris, Ivan. *The Riverside Puzzles.* New York: Walker & Co., 1969. 127 pp.

A collection of 50 or more word puzzles, stick games, logic problems, and mathematical puzzles.

Muller, Fritz. *Warum? Fröhliche Fragen zum Nachdenken.* Leipzig: Staackmann, 1926.

Papin, Maurice. *Colles et astuces mathématiques.* Paris: Blanchard, 1972. 163 pp.

A collection of problems and puzzles, appropriate for secondary school level; some well known, some new.

Perleman, Ya. I. *Zanimatelnye zadachi i opiti.* Moscow, 1959.

Phillips, Hubert. *Something to Think About.* London: M. Parrish, 1958.

Ranucci, Ernest R. *Puzzles, Problems, Posers, and Pastimes.* Boston: Houghton Mifflin Co., 1972.

A series of three booklets, each containing problems at a different level of difficulty; 75 problems in all.

Rosenberg, Nancy. *How to Enjoy Mathematics with Your Child.* New York: Stein & Day, 1970. 186 pp.

Figurate numbers, magic squares, intuitive topology, flexagons, paper folding, and so on.

Sackson, Sidney. *A Gamut of Games.* New York: Random House, 1969. 224 pp.

Comprehensive and authoritative.

Scripture, Nicholas E. *Puzzles and Teasers.* New York: Van Nostrand Reinhold Co., 1970. 74 pp.

A brief collection of simple puzzles involving elementary mathematics.

Silverman, David L. *Your Move.* New York: McGraw-Hill Book Co., 1971. 221 pp.

Puzzles dealing with cards, chess, number games, and so on, with emphasis on game strategy and decision making.

Souza, Júlio. *Matemática divertida e fabulosa; problemas curiosos anedotas, recreaçõs geométricas, etc.* São Paulo: Edicão Saraiva, 1962.

Sperling, Walter. *Die Grübelkiste; ein Buch zum Kopfzerbrechen.* Zurich: A. Müller, 1953.

Taylor, Judith M. *Fun with Mathematics.* Oxford: Basil Blackwood, 1972. 32 pp.

Simple recreations for the elementary school level.

Ulam, Stanislaw. *Problems in Modern Mathematics.* New York: John Wiley & Sons, 1960, 1964. (Paper)

Webster, David. *Brain Boosters.* London: J. M. Dent & Sons, 1969. 94 pp.

Mostly science riddles; some puzzles concerning shapes and knots; junior high school level.

Yaglom, A. M., and I. M. Yaglom. *Challenging Mathematical Problems.* Vol. 1. San Francisco: Holden-Day, 1964.

APPENDIX B

Chronological Synopsis of Martin Gardner's Column in Scientific American

The well-known monthly column "Mathematical Games" by Martin Gardner in *Scientific American* does not always lend itself neatly to bibliographic listing. Much of this material has appeared subsequently in book form, and many of the items are listed in volumes 1 and 2 of the present *Bibliography of Recreational Mathematics*. Most of the more recent articles are given here in volume 3 under appropriate subheadings. Nevertheless, for the reader's convenience we append a complete list of titles, in essential form, dating from December 1956 (the column's inception) to February 1973, inclusive.

NOTE. Based in large part on a list compiled by James A. Dunn in *Mathematics Teaching*, no. 52, pp. 59–60; Autumn 1970. By courtesy of the author and editor.

Dec	56	Flexagons
Jan	57	Magic matrices
Feb	57	Nine problems
Mar	57	The game "Ticktacktoe"
Apr	57	Paradoxes
May	57	Games: Icosian; Tower of Hanoi; polyominoes
June	57	The Möbius band
July	57	The game of HEX
Aug	57	Sam Loyd
Sept	57	Card tricks
Oct	57	Mnemonic devices
Nov	57	Nine puzzles
Dec	57	Polyominoes
Jan	58	Fallacies
Feb	58	The game of Nim
Mar	58	Left and right handedness

Apr	58	The monkey and the coconuts
May	58	Tetraflexagons
June	58	The puzzles of H. E. Dudeney
July	58	Number tricks
Aug	58	Nine brainteasers
Sept	58	The Soma cube
Oct	58	Four mathematical diversions involving topology
Nov	58	Perfect squares and perfect rectangles
Dec	58	Diversions which involve the Platonic solids
Jan	59	Mazes: how they can be traversed
Feb.	59	Brainteasers that involve formal logic
Mar	59	Magic squares
Apr	59	Problems
May	59	Nine brainteasers
June	59	The game of Eleusis
July	59	Origami
Aug	59	The golden ratio (Phi)
Sept	59	Mechanical puzzles
Oct	59	Probability
Nov	59	Graeco-Latin squares
Dec	59	Group theory: diversions that clarify
Jan	60	Numerology (Dr. Matrix)
Feb	60	Brainteasers
Mar	60	The games and puzzles of Lewis Carroll
Apr	60	Board games
May	60	The packing of spheres
June	60	Paperfolding and papercutting
July	60	"PI"
Aug	60	Magic tricks based on mathematical principles
Sept	60	The four-color problem
Oct	60	Nine brainteasers
Nov	60	More about polyominoes
Dec	60	Some recreations based on the binary system
Jan	61	Numerology (Dr. Matrix)
Feb	61	The ellipse
Mar	61	MacMahon's cubes and dominoes
Apr	61	Coxeter's *Introduction to Geometry*
May	61	Tricks and puzzles
June	61	Brainteasers
July	61	Board-games
Aug	61	Calculus of finite differences
Sept	61	Topological diversions
Oct	61	The exponential constant
Nov	61	Dissections
Dec	61	Probability and gambling
Jan	62	The fourth dimension
Feb	62	Eight problems
Mar	62	How to build a game-learning machine
Apr	62	Three types of spirals and how to construct them

May	62	Symmetry and asymmetry
June	62	The game of solitaire
July	62	Abbot's "Flatland" and two-dimensional geometry
Aug	62	Tricks collected at a fictitious magicians' convention.
Sept	62	Tests of division
Oct	62	A collection of nine puzzles involving numbers, logic and probability
Nov	62	Checker-board puzzles: dissections, etc.
Dec	62	Manipulations with strings
Jan	63	Numerology (Dr. Matrix)
Feb	63	Curves of constant width
Mar	63	Paradoxes
Apr	63	Foolishness for April Fools' Day
May	63	Reptiles
June	63	Helical structures: spirals and corkscrews
July	63	Topological diversions
Aug	63	Perms and paradoxes in combinatorial mathematics
Sept	63	How to solve puzzles by graphing the rebounds of a bouncing ball
Oct	63	Four board games
Nov	63	Nine problems
Dec	63	Parity tests: odd and even
Jan	64	Numerology (Dr. Matrix)
Feb	64	Sliding puzzles: the 15 puzzle
Mar	64	Prime numbers
Apr	64	Planar graphs: sets of vertices connected by edges
May	64	Number bases: the false coin problem
June	64	Nine short problems and more about primes
July	64	Curious properties of a cycloid curve
Aug	64	Magic tricks based on mathematical principles
Sept	64	Word games: puns, palindromes, etc.
Oct	64	Simple proofs of Pythagoras
Nov	64	Infinite series and the concept of limit
Dec	64	Polyiamonds
Jan	65	Numerology (Dr. Matrix)
Feb	65	Tetrahedrons
Mar	65	Nine short problems
Apr	65	The infinite regress: snowflake curves, etc.
May	65	The lattice of integers considered as an orchard or a billiard table
June	65	Postman problems: routing problems
July	65	"Op Art" patterns: tessellations
Aug	65	Communication with intelligent organisms in other worlds
Sept	65	The "Superellipse": a curve between the ellipse and the rectangle
Oct	65	Pentominoes and polyominoes
Nov	65	Nine elementary word and number problems
Dec	65	Magic stars, graphs and polyhedrons
Jan	66	Numerology (Dr. Matrix)
Feb	66	Coin puzzles
Mar	66	The hierarchy of infinities
Apr	66	The eerie mathematical art of Maurits C. Escher
May	66	How to "cook" a puzzle, or mathematical one-uppery

June	66	Efforts to trisect the angle
July	66	Wilhelm Fliess and his theory of male and female life cycles
Aug	66	Twenty-three problems solvable by reasoning based on elementary physical principles
Sept	66	The problem of Mrs. Perkin's quilt
Oct	66	Can the shuffling of cards be reversed?
Nov	66	To visualize a four-dimensional figure
Dec	66	Pascal's triangle
Jan.	67	"Acrostics" (Dr. Matrix)
Feb	67	Mathematical strategies for two person contests
Mar	67	Eight problems solvable with elementary techniques
Apr	67	Professional mental calculators
May	67	Cube-root extraction and the calendar trick
June	67	Polyhexagons and polyabolos
July	67	"Sprouts" and "Brussels sprouts": topological games
Aug	67	In which the computer prints out mammoth polygonal factorials
Sept	67	Double acrostics
Oct	67	Problems on the knight's move in chess
Nov	67	Nine logical and illogical problems to solve
Dec	67	Game theory is applied (for a change) to games
Jan	68	Numerology (Dr. Matrix)
Feb	68	Tree graphs and forests of trees
Mar	68	Perfect and amicable numbers
Apr	68	Puzzles and tricks with a dollar bill
May	68	Packing: circles and spheres
June	68	Combinatorial possibilities in a pack of shuffled cards
July	68	Random numbers
Aug	68	Thirty-one quick problems
Sept	68	Counting systems and the relation between numbers and the real world
Oct	68	MacMahon's color triangles
Nov	68	The ancient lore of dice
Dec	68	The Möbius strip
Jan	69	Numerology (Dr. Matrix)
Feb	69	Boolean algebra, Venn diagrams and the propositional calculus
Mar	69	The Fibonacci sequence
Apr	69	Eight problems emphasizing gamesmanship, logic and probability
May	69	Random walks
June	69	Random walks on the square and the cube
July	69	Tricks, games and puzzles with matches
Aug	69	Simplicity as a scientific concept
Sept	69	Constructions with a compass and a straightedge: with a compass
Oct	69	Matrix of the lunar flight of Apollo 11 (Dr. Matrix)
Nov	69	"Patterns," a new paper and pencil game based on inductive reasoning
Dec	69	Dominoes: a handful of combinatorial problems
Jan	70	The abacus: primitive but effective digital computer
Feb	70	Nine new puzzles to solve, some answers and addenda
Mar	70	Cyclic numbers and their properties
Apr	70	Some mathematical models embedded in the solar system
May	70	Optical illusions

June	70	Elegant triangle theorems
July	70	Diophantine analysis and Fermat's "last theorem"
Aug	70	Backward run numbers, letters, words and sentences
Sept	70	On the cyclical curves generated by rolling wheels
Oct	70	Solitaire game of "Life"
Nov	70	New collection of short problems
Dec	70	The paradox of the nontransitive dice
Jan	71	Lessons from Dr. Matrix in chess and numerology
Feb	71	On cellular automata, self-reproduction, the Garden of Eden and the game of "Life"
Mar	71	The orders of infinity, the topological nature of dimension and "supertasks"
Apr	71	Geometric fallacies: hidden errors pave the road to absurd conclusions
May	71	The combinatorial richness of folding a piece of paper
June	71	The Turing game and the question it presents: can a computer think?
July	71	Quickie problems: not hard, but look out for the curves!
Aug	71	Ticktacktoe and its complications
Sept	71	The plaiting of Plato's polyhedrons and the asymmetrical Yin-Yang-Lee
Oct	71	New puzzles from the game of Halma, the noble ancestor of Chinese checkers
Nov	71	Advertising premiums to beguile the mind: classics by Sam Loyd, master puzzle-poser
Dec	71	Further encounters with touching cubes, and the paradoxes of Zeno as "supertasks"
Jan	72	How to triumph at Nim; the Hackenbush game
Feb	72	Dr. Matrix proposes some heteroliteral puzzles
Mar	72	The graceful graphs of Solomon Golomb
Apr	72	A topological problem, and eight other puzzles
May	72	Challenging chess tasks for puzzle buffs
June	72	A miscellany of transcendental problems
July	72	Amazing mathematical card tricks
Aug	72	The binary Gray code and puzzle solving
Sept	72	Pleasurable problems with polycubes
Oct	72	The long arm of coincidence
Nov	72	Sir Francis Bacon's biliteral cipher
Dec	72	Knotty problems with a two-hole torus
Jan	73	New games: Sim, Chomp, and Race Track
Feb.	73	Up and Down Elevator games; Piet Hein's mechanical puzzles
Mar	73	The calculating rods of John Napier

Glossary

The following selected list of 500 entries contains many, if not most, of the terms commonly encountered in the literature of recreational mathematics. Some of the terms are doubtless familiar to the reader, but others may not be so widely known. For some of the more technical terms, a simple basic explanation is offered rather than a precise mathematical definition.

The number of mathematical "games" with identifiable names (for example, Nim, Oware, Reversi) has been judiciously held to a minimum, for new ones seem to sprout every day.

In some instances a brief annotation indicates the presumed originator of the term or the place where the term probably first appeared in print. Much as it might have been desirable, it became unfeasible to do this systematically throughout.

Insofar as the writer is aware, no such extensive glossary has ever been compiled. As a "first approximation," may it serve the reader well.

Abacus. A mechanical device used to facilitate arithmetical computation. In Roman times it took the form of a dust board with counters or a grooved table with beads. In more modern times it was commonly fashioned in the form of a rectangular frame with parallel wires on which an appropriate number of beads might slide, the wires serving as positional-value markers. In China the abacus is known as the *suan-pan;* in Japan, the *soroban;* in Russia, the *s'choty.*

Abundant number. Any integer the sum of whose divisors, excluding the given integer, exceeds the number itself. Thus 18 is an abundant number, since $1 + 2 + 3 + 6 + 9 > 18$. Every multiple of a perfect number (excluding the first multiple) or of an abundant number is an abundant number.

Abundant numbers are also known as *excessive* or *redundant* numbers (q.v.).

Acrostic. A series of printed lines or verses in which the first, last, or other particular letters from a meaningful word, phrase, sentence, or name.

Afghan bands. Another name for Möbius bands; sometimes used by professional magicians when suitably adapted. See *Möbius band.*

Algebraic magic square. Any even magic square in which the sum of the numbers in every quadrant of the square equals the magic constant.

Algorism. An earlier term, now replaced by the term *algorithm.* In medieval times, *algorism* pertained specifically to positional notation used with Arabic numerals and a decimal-numeration system.

Algorithm. Any particular procedure for solving a given type of problem; or, any specific method used to carry out a computation.

Aliquot divisors. The aliquot divisors of an integer comprise all its integral divisors, including unity, but excluding the integer itself; synonymous with *proper divisors.*

Allomorph. As sometimes used in crystallography or geometry, an allomorph is a polyhedron having the same Eulerian description (V_n, F_n, E_n) as another polyhedron but differing from it in the types of polygons that make up its faces.

Alphametic. Any cryptarithm that employs letters in place of digits, with these letters forming related words or meaningful phrases. [J. A. H. Hunter, 1955.]

Amicable numbers. Any two numbers N_1 and N_2 such that the sum of the proper divisors of N_1 equals N_2 and the sum of the proper divisors of N_2 equals N_1. Thus 220 and 284 constitute a pair of amicable numbers, since $S(220) = 284$ and $S(284) = 220$, where $S(N)$ represents the sum of the divisors of N, exclusive of N itself.

An alternative definition is that two numbers are amicable if their sum is the sum of *all* the divisors of either of the numbers.

A "chain" of numbers is said to be amicable if each is the sum of the proper divisors of the preceding number, the last being considered as preceding the first of the chain. Amicable number triples, quadruples, quintuples, and k-tuples have also been defined. [*A.M.M.* 20:84; 1913.]

Anabasis. An old board game similar to Chinese checkers.

Anaglyph. A composite picture or diagram printed in two colors, usually blue and red, such that a three-dimensional image is seen when viewed through spectacles having lenses of corresponding colors.

Anagram. The transposition of the letters of a word or sentence to form a new word or sentence. Also, a word-building game.

Anallagmatic pavement. A variety of pavement made with square tiles of two colors so arranged that when any two rows or any two columns are placed together side by side, half the cells next to one another are of the same color and half are of different colors. [J. J. Sylvester, 1868.]

Anchor ring. See *Torus*.

Annulus. The area included between two concentric circles. The area of an annulus is given by $A = \pi (R^2 - r^2)$, where R and r are the radii of the larger and smaller circles, respectively.

Antimagic squares. An $n \times n$ square array of integers from 1 to n such that each row, column, and principal diagonal produces a different sum and these sums form a scrambled sequence of consecutive integers. [*R.M.M.*, no. 7, p. 16; Feb. 1962.]

Antipalindromic number. An integer in which each digit differs from the corresponding digit of its reverse, as in 17683492; it must have an even number of digits. Its coincidence ratio is zero.

Antisnowflake curve. Formed in the same way as in the snowflake curve, only the equilateral triangles are turned inwards instead of outwards.

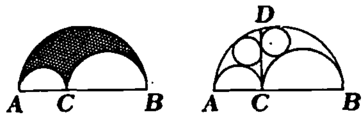
Antinomy. A logical contradiction, such as between two statements or laws both of which are assumed to be true; or, the contradiction arising between the conclusions correctly derived from two such statements. In common practice, *antinomy* and *paradox* are regarded as synonymous, although strictly speaking, the term *paradox* is also correctly used in a broader sense.

Antiprism. A prismatic polyhedron whose two bases, although parallel, are not similarly situated, but each vertex of either corresponds to a side of the other so that the lateral edges form a zig-zag; also known as a *prismoid*.

Apeirogon. A degenerate polygon, that is, the limiting form of a p-gon, as p approaches infinity, and hence an infinite line broken into segments.

Apollonian problem. A classic problem of antiquity that required the construction of a circle or circles tangent (internally or externally) to three given circles. Depending on the original given configuration, there may be as many as eight required circles or there may be none.

Arbelos. A geometric configuration attributed to Archimedes: also known as the *cobbler's knife* or the *sickle of Archimedes.* It is bounded by three semicircles tangent to each other at their extremities. The arbelos in the figure shown has the same area as the circle having $\overline{CD}$ as a diameter. The segment $\overline{CD}$ divides the arbelos into two parts, whose inscribed circles are equal.



Arc. A route in a graph that passes through no vertex more than once.

Archimedean solids. These are semiregular polyhedra, that is, "facially" regular, which means that every face is a regular polygon, although the faces are not all of the same kind; however, the faces are arranged in the same order around each vertex. There are exactly thirteen Archimedean solids, two of which occur in two (enantiomorphic) forms. See also *Uniform polyhedrons.*

The term *semiregular* may appropriately be applied to both facially and vertically regular polyhedra, but it is often used exclusively of the former. [L. Lines, *Solid Geometry*, 1935.]

These semiregular polyhedra may also be extended to include stellated forms of Archimedean solids with star faces or star vertices, or both.

Asymmetric. The condition of being identical on both sides of the central line of symmetry but in "reverse" order; a mirror image kind of relation.

Automorph. Any integer expressible in only one way in the form $x^2 + Dy^2$ or $x^2 - Dy^2$ is called a *monomorph*; if it is so expressible in more than one way, it is called a *polymorph*. Both forms are known as *automorphs*.

More technically, an integral transformation of determinant unity that leaves q unaltered is called an automorph of q, where by q is meant a form such as $x^2 + y^2$, and so on. [L. E. Dickson, *Introduction to the Theory of Numbers*, p. 72.]

Automorphic numbers. The class of those integers with the property that the squares of the last n digits are the same as those of the number itself; for example, $25^2 = 625$; $76^2 = 5{,}776$. Again, the square of any number ending in 625 also ends in 625; for example, $(625)^2 = 390{,}625$; $(2{,}625)^2 = 6{,}890{,}625$.

Ball-piling. Refers to the possible ways of arranging a number of small equal spheres in horizontal layers to fill a rectangular box.

Betti number. The Betti number of a surface is a topological invariant that gives the maximum number of cuts that can be made without dividing the surface into two separate pieces.

Bicimals. A term sometimes used to designate binary decimals.

Bifactorials. Bifactorial n, written as $n!!$, is defined as follows:

$$n!! = 1! \cdot 2! \cdot 3! \cdot \ldots \cdot (n-1)!\, n!$$

For example, $4!! = 1!\ 2!\ 3!\ 4! = (1)\ (2)\ (6)\ (24) = 288$.

Bigrade. A multigrade that holds only for $n = 1,2$.

Bimagic square. A magic square is bimagic if the square formed by replacing each of its numbers by its second power is also a magic square.

Binary games. Recreational puzzles or games (such as Nim) that involve the binary scale of notation in their solution.

Binary numeration. A numeration system having base two, or on the scale of two, and so requiring only two digits, viz., 0 and 1. Thus:

Base 10	*Base 2*
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011
12	1100
.	.
.	.
.	.

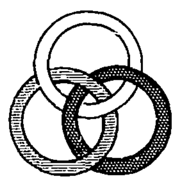
Binary numeration is used in solving certain weight problems, in explaining peasant multiplication, and in playing the game of Nim.

Bishop's re-entrant path. Similar to a knight's re-entrant tour, referring of course to the bishop. See *Knight's tour*.

Black. A topological pencil-and-paper game played on a checkered field, usually 8×8. [William Black, 1960.]

Bordered magic square. A magic square that contains within itself one or more other magic squares, revealed by successively stripping off each border of cells.

Borromean rings. A unique interlacing of three rings such that no two of the rings are linked, and yet the arrangement cannot be separated; however, if any single ring is broken and removed, the two remaining rings will be found to be unconnected.



Boss puzzle. See *Fifteen puzzle.*

Bracelet. One period of a simple periodic series considered as a closed sequence with terms equally spaced around a circle. Hence a bracelet may be regenerated by starting at any arbitrary point and applying the generating law. [*A.M.M.* 74:769; June 1967.]

Brachistochrone. The curve between two points that is traced in the shortest time by a body moving under an external force without friction; the curve of quickest descent.

Bridg-it. A topological game using a board of thirty black and thirty red spots placed in alternate rows. Two players, using black and red pencils, connect pairs of black and red spots, respectively, without crossing lines. [David Gale, 1958.]

A winning strategy has been determined, thus making it useless as a fair game. [Martin Gardner, *New Mathematical Diversions from Scientific American*, pp. 212–13.]

Brussels sprouts. A modification of the game of Sprouts (q.v.).

Buffon's needle problem. In its original form (1733), the problem was, Given a needle of length a and an infinite grid of parallel lines with a common distance d between them, what is the probability $P(E)$ that the needle, when tossed at the grid randomly, will cross one of the parallel lines? A modern generalization asks, How many lines might we expect the needle to cross? The answer in the first case is $P(E) = \frac{a}{\pi d}$, where $a \gtreqless d$; in the second case, $e(N) = \frac{2a}{\pi d}$.

Bulo. The name applied in Denmark to the game of *Tac Tix*.

Cabala. A system of occult interpretation of the Scriptures among Jewish rabbis and some medieval Christians; hence a Cabalist was a person engaged in mystic arts, including numerology, gematria, and so on.

Calculating prodigies. Individuals manifesting extraordinary powers of mental calculation. Their performances, although remarkable, in all probability reflect no "different kind" of mental abilities than those of others. Documented instances of calculating prodigies for the most part are those of young, illiterate, or uneducated persons with exceptionally good memory facilities who nearly always lost these powers in later life.

Calendar problems. Any problems or puzzles related to the calendar; commonly concerned with the determination of the date of Easter, the construction of a perpetual calendar, the probability of coincident birthdays, the occurrence of leap years or of Friday the 13th, and so on.

Cantometrics. A new field (ca. 1965) of activities that musicologists suggest deals with the relation of any culture's music with its social characteristics. Interest in this field was presumably stimulated by computerized music.

Cattle problem of Archimedes. A fantastic problem in which it is required to determine the number of white, black, spotted, and yellow bulls and the number of cows of the corresponding colors, given nine numerical conditions to be satisfied with regard to these eight variables. Analysis of the problem leads to the Pellian equation

$$y^2 - 410{,}286{,}423{,}278{,}424t^2 = 1,$$

an equation that has yet to be solved completely.

Charm. A side chain of a bracelet. [*A.M.M.* 74:769; June 1967.]

Cheery sequence. A sequence of integers, starting with any arbitrary integer, where each succeeding term is the sum of the squares of the digits of the previous terms. For example:

(A) 4, 16, 37, 58, 89, 145, 42, 20, 4, . . .

(B) 12, 5, 25, 29, 85, 89, 145, 42, 20, 4, 16, 37, 58, 89, . . .

[Donald C. Duncan, *M.T.* 65:627–29; Nov. 1972.]

Chess task. Refers to a specific objective to be reached; not to be confounded with a "task problem" in chess (q.v.).

Chinese checkers. A game played on a hexagonal-cell board that is generally shaped like a six-pointed star.

Chinese rings. A recreational toy consisting of several rings hung on a bar in such a way that the ring (or first two rings) at one end can be taken off or put on the bar at pleasure; but any other ring can be taken off or put on only when the one next to it (towards the end ring) is on and all the rest are off. Only one ring can be taken off or put on at a time, and the order of the rings cannot be changed.

Chromatic graph. A complete graph (in a plane or in 3-space) whose edges are colored either red or blue; then a monochromatic triangle is one whose three sides are of the same color.

Chromatic number. A topological invariant for a given surface, the chromatic number is the maximum number of regions that can be drawn on the surface in such a way that each region has a border in common with every other region. Thus if each region is assigned a different color, each color will border on every other color. Hence the term *chromatic number* is also used to designate the minimum number of colors sufficient to color any finite map on a given surface.

Cipher. See *Cryptogram.*

Circuit. An arc that returns to its starting point, that is, a route that revisits only the beginning vertex.

Circuit rank. The circuit rank of a particular graph G is the number of edges of G minus the number of vertices of G plus one.

Circulating decimal. See *Repeating decimal.*

Clock solitaire. A solitaire type of card game in which the fifty-two cards of a deck are dealt into thirteen face-down piles of four cards each, arranged like the numerals of a clock, with one pile in the center.

Close packing. The arranging of equal circles (or equal spheres) so that they are inscribed in regular tessellations. Such packings vary in density, depending on the tessellation used.

Code. The system or key used in preparing a cryptogram.

Coincidence ratio. The ratio of the number of coincidences (agreements) between the digits of an integer and the digits of its reverse to the number of digits in the integer. Thus the coincidence ratio of 1437245 is 3/7.

Collapsible compasses. See *Euclidean compasses.*

Colored cubes (and squares). A variety of recreations with colored squares and cubes, ranging from Major MacMahon's tiles and cubes to puzzles such as Instant Insanity.

Combination. Any particular selection of one, several, or all of the elements of a finite set of entities, irrespective of the order of selection.

Compasses. Modern compasses, that is, those that do not collapse when opened and may thus be used as a divider to transfer a distance as well as to draw a circle. It can be shown that any geometric construction that can be effected with the straightedge and modern compasses can also be performed with the straightedge and Euclidean compasses; however, the converse is not true. See also *Euclidean compasses.*

Complete graph. A graph of n vertices with edges connecting all pairs of vertices, that is, with $\frac{1}{2}n(n - 1)$ edges.

Composite number. Any integer that is composed of two or more proper factors or divisors, not necessarily different. Examples: $8 = 2 \cdot 2 \cdot 2$; $14 = 2 \cdot 7$; $18 = 2 \cdot 3 \cdot 3$; $100 = 10 \cdot 10$. The number 1 is regarded as neither composite nor prime.

Any composite number can be expressed as the product of prime numbers in one and only one way, disregarding the order in which the factors are stated. For example: $105 = 3 \cdot 5 \cdot 7$; $360 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 2^3 \cdot 3^2 \cdot 5$.

Concentric magic squares. See *Bordered magic squares.*

Congruent numbers. A number k is called congruent if integers x and y exist such that both $x^2 + ky^2$ and $x^2 - ky^2$ are perfect squares. An example of a possible solution for the smallest congruent number 5 is the following: $(41)^2 + 5(12)^2 = (49)^2$ and $(41)^2 - 5(12)^2 = (31)^2$.

Connected graph. A graph in which every vertex is connected to every other vertex by some arc.

Conservative number. A number that has the property of dividing evenly into its reverse; for example, $9801/1089 = 9$. Or, a number in which the digits are "conserved" after some operational change or transformation. The term is not as yet universally recognized.

Constant-width curves. If a closed convex curve is placed between two parallel lines and the lines are moved together until they are tangent to the curve, the distance between the parallels is called the width of the curve. If a curve (e.g., the circle) has the same width in all directions, it is a curve of constant width. There are infinitely many such curves.

Contack. A game played with equilateral-triangle tiles. [Martin Gardner, *Sci.Am.* 204:173; Mar. 1961.]

Cosmograph. A visual pattern produced by interfering sound waves, somewhat similar to Lissajous's figures. These infinitely varied optical pictures of sound waves are easily related to modern-art forms.

Cross-number puzzle. Similar to crossword puzzles, with numbers instead of words to be identified.

Crossed ladders. A perennial geometric problem involving two ladders of known but unequal lengths crossing so that their feet are respectively at the bases of two walls encompassing an alley. The usual question is, If the height above the pavement of the point at which the ladders cross is known, find the width of the alley.

Crossings, problems of. Problems of difficult crossings, such as the dilemma of the boatman with the wolf, the cabbage, and the goat, or the problem of the jealous husbands and their wives trying to cross a river in a boat that can hold only two persons.

Cryptanalysis. The art or science of solving a cryptogram, that is, discovering the true meaning of a "secret" or coded message without the knowledge of the code or key used in preparing the cryptogram; not to be confounded with the practice of deciphering a cryptogram, which refers to "translating" it with the aid of an appropriate code.

Cryptarithm. An indicated arithmetical operation in which some or all of the digits have been replaced by letters or symbols and where the restoration of the original digits is required. Embraces *alphametics*, *faded documents*, and *skeleton divisions* (q.v.). [*Sphinx* 1:50; 1931.]

Cryptogram. A "secret" or concealed message consisting of specific letters, numerals, or other symbols used in connection with some prearranged system (key or code) that permits of conveying a meaning other than the apparent message. Cryptograms are also called *ciphers*.

Cryptography. The art of composing secret messages that are intelligible to those who are in possession of the key, or code, and unintelligible to all others. The act of writing such a secret message is called *encoding* it; the act of interpreting it with the aid of a key is called *decoding*, or deciphering.

Curve of error. See *Probability curve.*

Curve stitching. The art of creating designs consisting of straight-line envelopes made with colored threads stitched on cards in accordance with some preassigned pattern of punched holes. [Edith Somervell, London, 1906.]

Cyclic decimal. See *Repeating decimal.*

Cyclic number. An integer of n digits with the unusual property that when multiplied by any number from 1 through n the product contains the same n digits as the original number in the same cyclic order. The smallest cyclic number is 142,857.

Cyclic path. A path that returns to its starting point.

Cyclomatic number. Another name for the circuit rank of a graph, that is, the number of edges of the graph minus the number of vertices of the graph plus one.

Cyclotomic equation. The equation $x^{p-1} + x^{p-2} + x^{p-3} + \cdots + 1 = 0$, obtained by dividing $x^p - 1 = 0$ by $x - 1$, where p is prime. The cyclotomic equation is irreducible, and its roots are on a circle.

Decanting problems. Also known as pouring problems, jug problems, and Tartaglia's measuring problems. The problem is how to transfer a given quantity of liquid from one container to another, using only a definite number of specific measuring vessels for the purpose.

Dee-dee consecutives. If one number divides another, and the dividend D suggests a consecutive extension of the divisor d, then we have a $d|D$ consecutive, that is, d divides D. For example: $1|2$; $2|3456$; $3|456789$; $5|43210$; $34|3536$; $7|800009$.

Defective number. *See Deficient number.*

Deficient number. Any integer the sum of whose integral divisors, excluding the given integer, is less than the integer itself. Thus 10 is a deficient number, since $1 + 2 + 5 < 10$. Or, alternatively, a deficient number is one that is greater than the sum of its proper divisors; same as *Defective number.* Every proper divisor of a perfect number is deficient.

Delian problem. See *Duplication of the cube.*

Denary numeration. A numeration system using base ten, or on the scale of ten, and requiring the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Also called *decimal notation.*

Deployment. A pencil and paper game for two players, played on a 5×5 array of 25 squares; similar to ticktacktoe but considerably more sophisticated. [William H. McGrail, Worcester, Mass.]

Derangement. If $(a_1, a_2, a_3, \cdots, a_n)$ is a permutation of n elements labeled $1, 2, 3, \cdots, n$, then the permutation $(a_1, a_2, a_3, \cdots, a_n)$ is a derangement if $a_i \neq i$ $(i = 1, 2, 3, \cdots, n)$. Thus a derangement has no element in its natural position. [H. J. Ryser, *Combinatorial Mathematics* (Carus Monograph no. 14), 1963.]

Diablotin. See *Fifteen puzzle.*

Diabolic doughnut. A panmagic square, first rolled into a cylinder, then bent into a torus; all the rows, columns, and diagonals are closed loops, but the diabolic properties still hold.

Diabolic hypercube. The two-dimensional projection of the hypercube (or tesseract) lends itself to magic-square arrangements.

Diabolic magic square. Same as a panmagic square; also called *pandiagonal* and *Nasik* squares.

Diamond. A polyiamond consisting of two triangles.

Dido's problem. To find the curve, with a given perimeter, that encloses the maximum area; in general, it is a circle. If a portion of the boundary is to be an arbitrary straight-line segment, then the required curve is a semicircle.

Difficult crossings. See *Crossings.*

Digit. In the base-ten system, any one of the ten Hindu-Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. In general, the individual symbols used in any cipherized system of numeration.

Digital diversions. The manipulation of numbers in such a way as to use their digits once only in order to arrive at a certain result, usually an identity. In general, the customary arithmetical operations and symbols are allowed, although in some instances the use of radicals, exponents, and factorials may also be admitted.

Digital invariant. An integer that is equal to the sum of the nth powers of its digits, or an integer for which the sum of the nth power of its digits is equal to some particular number. An example of the former is $153 = 1^3 + 5^3 + 3^3$.

Digital root (of an integer). The smallest positive integer to which the given integer is congruent modulo 9. It may be obtained by casting out 9s (replacing a final 0 with 9) or more laboriously by summing the digits of the integer, summing the digits of that sum, and continuing the process until a single digit is obtained. In any base b, use $b - 1$ in place of 9. [H. E. Dudeney, *Amusements in Mathematics*, p. 157; 1917.]

Digital sum. The digital sum of a number is the sum of the digits of its numerals. For example, the digital sum of 124 is $1 + 2 + 4 = 7$; or, the "first" digital sum of 79 is $7 + 9 = 16$, whereupon these digits are again added, giving a final digital sum of $1 + 6 = 7$.

Dim. A three-dimensional version of the game of Sim. [Douglas Engel, *J.R.M.* 5:274; Oct. 1972.]

Diophantine equation. An indeterminate polynomial equation in two or more variables for which the desired solution values are to be either rational or integral numbers, usually the latter; for example, $x + 2y = 13$, or $a^2 + b^2 = c^2$.

Dissection problems. Geometric dissections involve the cutting up of a geometric figure in accordance with some specified goal. More specifically, such problems call for the conversion of one figure to another by directly cutting it into a finite number of pieces and then rearranging them to form the other figure.

Divine proportion. See *Golden section.*

Dodecahedron. A convex dodecahedron is a convex solid bounded by twelve congruent pentagons; it has 12 faces, 20 vertices, and 30 edges.

Domino. A two-square polyomino. Also, familiar rectangular pieces divided into congruent squares bearing up to as many as six (or nine) pips in all possible combinations.

Doubly true addition. A cryptarithm or alphametic in which the addition of both the words and the numerals is true. [*A.M.M.* 68:1006; Dec. 1961.]

Dual solids. The dual of a polytope is another whose planes correspond to the vertices of the original figure and vice versa.

Duodecimal system. A system of numeration using base twelve and twelve digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E. Thus:

$$
\begin{aligned}
130_{(\text{twelve})} &= 1(12)^2 + 3(12)^1 + 0(12)^0 = \\
&\quad 144 + 36 + 0 = 180_{(\text{ten})} \\
2ET_{(\text{twelve})} &= 2(12)^2 + 11(12)^1 + 10(12)^0 = \\
&\quad 288 + 132 + 10 = 430_{(\text{ten})} \\
19E_{(\text{twelve})} &= 1(12)^2 + 9(12)^1 + 11(12)^0 = \\
&\quad 144 + 108 + 11 = 263_{(\text{ten})}
\end{aligned}
$$

Duplication of a cube. One of the three famous problems of antiquity—to construct a cube having twice the volume of a given cube, using only the straightedge and compasses. Known also as the *Delian* problem, it is impossible to solve under the given conditions, since it is impossible to construct with Euclidean tools a segment whose length is a root of a cubic equation with rational coefficients but with no rational root.

Dynamic symmetry. Refers to the classic art periods of Egypt and Greece, in which designs commonly involved ratios of $1:\sqrt{2}$, $1:\sqrt{3}$, and $1:\sqrt{5}$. These dynamic designs, so designated by Jay Hambidge, evolved by the use of areas rather than by line measurements.

Edge. A piece of a curve connecting two vertices of a graph and containing no other vertex.

Eleusis. A card game for three or more players, invented by Robert Abbott. It is played with a standard deck of playing cards and is characterized by its dependence on inductive thinking.

Ellipsograph. Known also as the *trammel of Archimedes;* a mechanical device used in drafting rooms for constructing an ellipse.

Elliptipool. Pool played on an elliptically shaped pool table; originally only a theoretical concept, but reportedly such a table has been built.

Enantiomorphic. A figure that is not reflexible is said to be enantiomorphic to its mirror image; for example, a pair of gloves.

Epimenides' paradox. A Cretan by birth, Epimenides said that all Cretans are liars, a statement that, if true, makes the speaker a liar for telling the truth.

Equidecomposable. Two figures are said to be equidecomposable if it is possible to decompose one of them into a finite number of parts that can be rearranged to form the second figure.

Euclidean algorithm. To find the highest common factor of two integers, divide the smaller into the larger, then divide the remainder into the preceding divisor; repeat this process until the remainder is zero. The last divisor used is the highest common factor.

Euclidean compasses. A pair of compasses such that if either leg is lifted from the plane, the instrument will automatically collapse. Thus the Euclidean compasses cannot be used for transferring a distance. Such an instrument, along with an unmarked straightedge, was indicated by the ancients when they spoke of "geometric construction," that is, using Euclidean tools. See also *Compasses.*

Euler circles. Similar to Venn diagrams; used not only to exhibit relations between sets but also to illustrate syllogistic reasoning.

Euler circuit. A cyclic path that covers each edge of a graph exactly once and returns to the starting point. See also *Unicursal curve.*

Euler line. A cyclic path that covers every edge of a graph.

Euler-Poincaré characteristic. A constant X, which defines a property of a surface:

$$X = V - E + F,$$

where V = the number of vertices, E = the number of edges, and F = the number of faces.

In a simply connected polyhedron (or connected map in a plane), the characteristic constant X = 2, yielding the familiar Euler formula $V - E + F = 2$.

Euler squares. An Euler square of order n is a square in which the cells are filled with n elements of one kind, $a_1, a_2, a_3, \cdots, a_n$, and n elements of another kind, $b_1, b_2, b_3, \cdots, b_n$, in such a way that—

1. each cell contains one element of each kind;
2. each element of the first kind is paired with each element of the second kind exactly once;
3. each row and each column contains all the elements of both kinds.

An Euler square may be regarded as a combination of two Latin squares. Euler squares are also known as *Graeco-Latin* squares.

Euler's theorem. This states that, topologically, for any map on a sphere, $V - E + F = 2$. The theorem also applies to convex polyhedrons; for example, a hexahedron, for which $V = 8$, $E = 12$, and $F = 6$.

Excessive number. An excessive number is any number that is less than the sum of its proper divisors; synonymous with *abundant* number and *redundant* number. For example, 12 is an abundant number, since $12 < 1 + 2 + 3 + 4 + 6$.

Factor. Any one of two or more numbers that are multiplied to form a product.

Factorials. If n is a natural number, then *factorial n* designates the product. $1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots \cdot (n-1)(n)$. This product is written as $\underline{|n}$ or $n!$, and is read "factorial n" or "n factorial." Factorial zero, $0!$, is arbitrarily defined to be unity.

Faded document. An indicated arithmetical operation in which some of the numerals have been rendered illegible, thus producing a cryptarithm.

Fairy chess. Any unconventional set of rules or nontraditional pieces used with some kind of a chessboard, as, for example, cooperative chess, four-handed chess, retrograde analysis, no checkmate, move-and-a-half, vertical-cylinder board, maximummer game, no capture chess, three- and four- dimensional chess, transportation chess, Möbius-strip chess, no-pawn chess, and so on.

Fallacies. A mathematical fallacy is an instance of reasoning that leads to a false or absurd conclusion. It may be due to a violation of a principle of logic, to a misleading diagram, to the denial of a previous assumption, or to the violation of a previously established definition, theorem, or principle.

Farey sequence. If all the proper fractions (written in lowest terms) having denominators not greater than a given integer are arranged in order of magnitude, the sequence is called a *Farey sequence*. In such a sequence, each fraction is equal to the fraction whose numerator is the sum of the two numerators on either side of it and whose denominator is the sum of the corresponding denominators. For example, for $N = 4$:

$$\frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{1}.$$

Fault-free rectangles. Any rectangular arrangement of interlocking dominoes such that there is no straight line, vertical or horizontal, that joins the opposite sides of the rectangular arrangement; for example, in the figure shown, $\overline{AB}$ is a fault line.

Feminine numbers. This term was used by the ancient Greeks to denote *even* numbers, which were regarded as ephemeral. Odd numbers were masculine; even numbers, which always contained other numbers, were femimine. Thus odd numbers were divine and heavenly, but even numbers were regarded as human and earthly. The number 2 was the first feminine number; the number 3 was the first masculine number. (The number 1 was the source of all numbers.)

Fermat numbers. Fermat numbers are numbers of the form

$$2^{2^n} + 1.$$

The first five Fermat numbers ($n = 0, 1, 2, 3, 4$) are 3, 5, 17, 257, and 65,537. Some Fermat numbers are prime; others are composite.

Fermat's last theorem. Fermat's statement was that $x^n + y^n = z^n$ has no solution in integers or rational numbers for $n > 2$. The "theorem" has not yet been proved.

Ferrying problems. See *Crossings.*

Fibonacci Nim. A modified form of Nim, for two players. Starting with a pile of n counters, the first player may not take the entire pile; thereafter either player may remove all remaining counters provided (1) that at least one counter is taken at each play, and (2) that neither player ever takes more than twice as many counters as his opponent took on his last play. [Robert E. Gaskell, ca. 1965; Martin Gardner, *Sci.Am.*, Mar. 1969, p. 119.]

Fibonacci numbers. The Fibonacci sequence $\{F_n\}$ is defined by $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Thus $\{F_n\} = \{0, 1, 1, 2, 3, 5, 8, 13, \cdots\}$.

Fifteen puzzle. This consists of a shallow square tray that holds exactly fifteen small square counters numbered from 1 to 15, with provision for a blank-square place. Initially placed in random order, the puzzle is to rearrange the fifteen squares in numerical order by sliding only, with the blank space finally remaining in the lower right-hand corner.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Figurate numbers. Figurate numbers are defined as the terms of certain arithmetic series and the terms of "sum-series" formed from these series. For example:

(A) $1, 2, 3, 4, 5, \cdots r$
(B) $1, 3, 5, 7, 9, \cdots (2r - 1)$
(A′) $1, 3, 6, 10, 15, \cdots r(r + 1)/2$
(B′) $1, 4, 9, 16, 25, \cdots r^2$

The terms of series such as (A′) and (B′) are called *plane figurate numbers*.

Finite geometry. Sometimes called a "miniature geometry"; any system of geometry based on the postulation of a finite number of points and a finite number of lines, where "points" and "lines" are not only undefined terms but admit of unconventional interpretations.

Flexagons. These are paper polygons specially folded from a strip of paper and having the unique property of changing exposed faces when they are "flexed." The original models were hexagonal in form with exposed triangular faces; other models, developed later, are made from square forms and expose square faces. Many varieties of both forms (hexaflexagons and tetraflexagons) have been developed, some of which are available on the market as toys under various trade names. [A. H. Stone, 1939.]

Flexahedron. Three-dimensional structures comprised of connected solids in such a way that they can be rotated or flexed in various ways.

Focus. A board game played with thirty-six counters, half of one color and half of another, on an 8×8 board from which three cells at each corner have been removed. [Sidney Sackson, in *Martin Gardner's Sixth Book of Mathematical Games from Scientific American*, p. 44; 1971.]

Four-color problem. The original map-coloring problem, which asked, Are four colors *sufficient* to color every possible map in the plane (or on the surface of a sphere) so that no two adjoining countries are of the same color? See also *Map coloring*.

Four-digits problem. To express with the digits 1, 2, 3, 4 the consecutive numbers from 1 upwards as far as possible, using each of the four digits once and once only in conjunction with arbitrarily chosen mathematical operations.

Four-fours problem. To express, using "ordinary" arithmetic and algebraic notation, the consecutive numbers from 1 upwards as far as possible in terms of four 4s.

Fourth dimension. A term referring alternatively to the "popular" aspects and to the mathematical considerations of n-dimensional space, where $n = 4$.

Franklin squares. An ingenious 8×8 magic square and another 16×16 square, both of which have unusual properties not possessed by other even-numbered magic squares. Both were created by Benjamin Franklin.

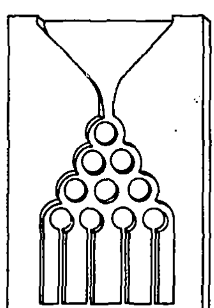
Freak cancellations. See *Illegal operations.*

Gale. A pencil and paper game that involves connecting black dots and colored dots, respectively; one set of dots is embedded in a similar rectangular array of the other set of dots. [Martin Gardner, *Second Scientific American Book of Mathematical Puzzles and Diversions*, p. 84; 1961.]

Galileo sequence. A sequence of the form

$$\frac{1}{3} = \frac{1+3}{5+7} = \frac{1+3+5}{7+9+11} = \cdots.$$

Galton probability board. A physical model illustrating the normal binomial distribution curve by allowing many small steel balls to drop over a series of rows of staggered pins, as shown in the figure. The probabilities of each ball falling to the left or to the right of two adjacent pins, or between them, are in the ratio 1:2:1; hence the probabilities for the entire board are proportional to the numbers of Pascal's triangle. (q.v.)



Galton quincunx. Same as the *Galton probability board.*

Gaussian curve. See *Probability curve.*

Gematria. A mystic pseudoscience based on the fact that the letters of various ancient alphabets (e.g., Greek, Hebrew) had numerical values and hence were used in computation.

Geoboard. A flat board into which nails or pins have been driven in a regular pattern. By slipping rubber bands around the pins, geometric figures and relations may be discovered. The pattern may be a square grid, a circular grid, or an isometric grid.

Geodesic. The shortest distance between two points on a surface. On a sphere, a geodesic is an arc of a great circle. Sometimes there are many equally short paths, as, for example, between two poles of a sphere.

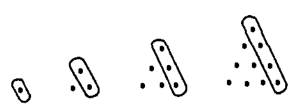
Geometric magic square. An $n \times n$ array of n^2 distinct integers with the property that the product of the n integers in any row, column, or main diagonal is equal to the same magic constant.

Glissette. The curve generated by a fixed point on a curve as the curve slides between given curves.

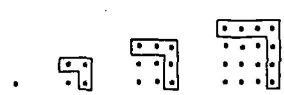
Gnomic magic square (third order). A 3×3 array in which the elements in each 2×2 corner have the same sum. [*M.Mag.* 43:70; Mar. 1970.]

Gnomic numbers. The successive numbers that, when added to triangles, squares, pentagons, and so on, produce an additional triangle, square, pentagon, and so on.

For triangles, the gnomic numbers are 1, 2, 3, 4, · · · .



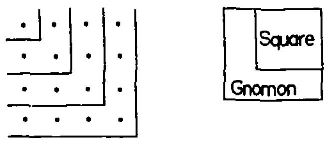
For squares, the gnomic numbers are 1, 3, 5, 7, · · ·.



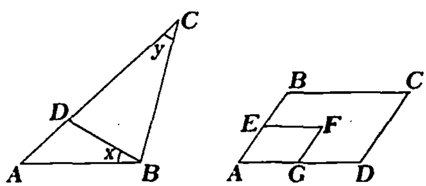
For pentagons, the gnomic numbers are 1, 4, 7, 10, · · ·.

Gnomon (1). In ancient Babylonia and Greece a gnomon was an astronomical instrument consisting of a vertical column or shaft for determining the altitude of the sun or the latitude of a position by measuring the length of the shaft's shadow cast at noon. By extension, it also refers to the raised part of a modern sundial that casts the shadow.

Gnomon (2). In connection with the theory of polygonal numbers, there is always a gnomon that can be added to a square number to create the next greater square number; for example, 5 is the gnomon of 4, 7 is the gnomon of 9, and so on. Or, conversely, the figure that remains of a square when a smaller square is cut off one corner.



Gnomon (3). In geometry, a gnomon is a portion of a geometric figure that has been added to (or taken away from) a figure so that the new figure (or remaining figure) is similar to the original figure. For example, in the figure shown $\triangle BCD$ is a gnomon to $\triangle ABD$ if $\angle y = \angle x$; or, $EFGDCB$ is the gnomon of $\square ABCD$.



Go. The national game of Japan; exceedingly complicated and difficult. The board consists of two sets of 19 parallel lines, mutually at right angles, forming 361 points of intersection. The game is played with 361 pieces called *stones* placed on these points of intersection; once a stone is placed on an intersection it remains there unless captured. The object of the game is to capture territory and pieces by surrounding them in certain ways.

Goldbach's conjecture. An unproved (by 1972) conjecture to the effect that every even integer (except 2) is the sum of two primes; for example, $28 = 11 + 17$, or $62 = 19 + 43$.

Golden rectangle. A rectangle whose sides are in the ratio of Φ:1; that is, approximately 8:5. Such a rectangle is said to be aesthetically the most pleasing rectangular shape; an ordinary postal card is very nearly this shape. See also *Phi*.

Golden section. A given line segment is said to be divided in extreme and mean ratio, or in the golden section, when the two parts a and b, are such that $a/b = b/(a+b)$, where $a < b$. See also *Phi.*

Go-moku. An Oriental board game played on a 19×19 Go-board. The number of stones is the same as in Go, although not all need to be used. The object of the game is to form a straight line of exactly five (not more) adjacent stones of one color.

Googol. A term coined by the late Edward Kasner; it represents the number 10^{100}. When written in full, it is seen as 1 followed by one hundred zeros.

Googolplex. Also coined by Kasner, a googolplex is 1 followed by a googol of zeros; that is, a googolplex is 10 raised to the googolth power, or $10^{(10^{100})}$.

Graeco-Latin square. Also known as Graeco-Roman square; another name for the Euler square (q.v.).

Graph. A figure consisting of points (vertices) and segments (edges) connecting some or all of these vertices. The edges may be straight or curved segments.

Graph theory. The term *graph* used in the context of graph theory, or network theory, refers to "linear graphs," that is, simple geometric figures consisting of points and lines connecting some of these points.

Halma. A game somewhat like Chinese checkers; played by two or four persons on a checkerboard with sixteen cells on each side. [Martin Gardner, *Sci.Am.*, Oct. 1971.]

Hamilton circuit. A circuit that covers all vertices of a graph.

Hamiltonian game. Determining along the edges of a regular dodecahedron the route that will pass once and only once through every vertex.

Happy integers. An integer is happy if and only if its cheery sequence has a period of 1. For example: 7 : 7 , 49, 97, 130, 10, 1, 1, 1, · · ·, [Donald C. Duncan, *M.T.* 65:627–29; Nov. 1972.] See *Cheery sequence.*

Harmonograph. An arrangement of swinging pendulums, frequently used to demonstrate simple harmonic motion. Suitably adjusted, it yields a variety of attractive patterns known as Lissajous's figures.

Hempel's paradox. A logical paradox according to which it can be shown that apparently no crows are black.

Heronian triangle. Any triangle whose sides are rational and whose area is also rational; also called a Heronian triple. There are infinitely many Heronian triangles; for example, (125, 136, 99) is a Heronian triple. Any Pythagorean triple is necessarily Heronian.

Heterosquare. An n by n array in which the $4n$ sums of the elements in the rows, columns, and diagonals (broken and unbroken) are all different. [*M.Mag.* 24:166; Jan. 1951.]

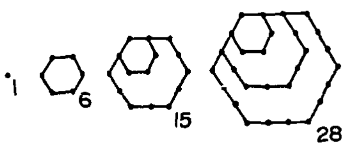
Hex. A game played on a board consisting of an array of congruent, interlocking hexagons, eleven on a side. [Invented independently by Piet Hein in 1942 and John F. Nash in 1948.]

Hexacube. The 166 pieces formed by joining six-unit cubes in all possible ways. [David Klarner.]

Hexadiangle. A board game played on a hexagonal board marked out in equilateral triangles. [G. Austwick, *M.Tchg.*, no. 59, p. 13; Summer 1972.]

Hexaflexagon. Six-sided paper structures that can be "flexed" so as to successively bring various surfaces to view; they are constructed by suitably folding a narrow strip of paper and can also be made effectively from strips of thin metal foil.

Hexagonal numbers. A class of polygonal numbers that may be defined by the geometric figure they represent; thus for a hexagon:



They may also be defined as sums of a special arithmetic sequence; thus $1 + 5 + 9 + 13 + 17 + \cdots + (4n - 3) = n(2n - 1)$.

Hexagram. The six-pointed starlike figure formed by two congruent equilateral triangles superposed so that their six vertices become the vertices of a regular hexagon.

Hexahedron. A convex hexahedron is a solid figure bounded by six plane polygons. There are only seven varieties of convex hexahedrons; one of these varieties is the regular hexahedron, or cube, which is bounded by six congruent squares, with 8 vertices, 6 faces, and 12 edges.

Hexatetraflexagon. A special species of tetraflexagon that can be flexed along both horizontal and vertical axes to expose all six of its square faces.

Hexiamond. A polyiamond consisting of six triangles.

Homeomorphic. If a geometric figure can be transformed into another by a topological transformation, the two figures are topologically equivalent, or homeomorphic. A topological transformation is a one-to-one correspondence between the points of two figures A and B such that open (or closed) sets in A correspond to open (or closed) sets in B. Any transformation that shrinks, expands, twists, and so forth, in any way without tearing, that is, a continuous transformation, is also a topological transformation.

Honeycomb. A three-dimensional honeycomb, or solid tessellation, is an infinite set of polyhedra fitting together in such a way as to fill all space exactly once, and such that every face of each polyhedron belongs to one other polyhedron. A honeycomb is regular if its cells are regular and equal.

Hopscotch. A modification of ticktacktoe played on a square array of nine lattice points, each player using three pieces; not to be confounded with the children's hopping game of the same name.

Howler. An amusing illegal (mathematical) operation; also known as a "lucky boner" or "making the right mistake."

Hypercube. A four-dimensional cube; also known as a tesseract. More generally, the term *hypercube* refers to any n-dimensional cube.

Icosahedron. A convex icosahedron is a convex solid bounded by twenty plane polygons. The regular icosahedron is bounded by twenty congruent equilateral triangles; it has 20 faces, 12 vertices, and 30 edges.

Icosian game. Same as the *Hamiltonian game.*

Illegal operation. An incorrect mathematical operation or algorithm innocently introduced into a calculation or proof but which nevertheless leads to a correct result. For example:

1. $\frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}$
2. $1^3 + 2^3 = 1 \times 3 + 2 \times 3 = 3 + 6 = 9$

Imbedded numerals. A two-digit numeral is said to be imbedded, for example, in a four-digit numeral if the digits of the former occur in any order in the latter. Thus in 2559 are imbedded 25, 29, 55, 59, 95, 92, and 52. [Mannis Charosh, *Mathematics Student Journal*, Problem 261; Jan. 1967.]

Indifference, principle of. Formerly known as the principle of insufficient reason, it states that if there are no grounds whatever for believing that any one of n mutually exclusive events is more likely to occur than any other, a probability of $1/n$ is assigned to each.

Inferential problems. Problems or puzzles of a logical nature rather than those involving computation or geometric configurations. For example: problems of difficult crossings; pouring problems; Smith-Jones-Robinson type of problems; the "unexpected egg" or the "unexpected hanging"; the colored hats or the smudges; truth and lying situations; and so on.

Infinite regress. An endless hierarchy of identical entities or operations, such as the reiterated images of a mirror reflected in a mirror or the segments of a curve of the snowflake type.

Integer. Any member of the set

$$\{\cdots, -4, -3, -2, -1, 0, +1, +2, +3, +4, \cdots\}.$$

Also, in the proper context, a natural number, a whole number, or a positive integer.

Integer, peak. An integer with digits that, reading from the left, steadily increase to a maximum and then steadily decrease to a final right-hand digit, as in 234631. [*J.R.M.* 4:170; July 1971.]

Integer, regular peak. A peak integer with digits on each slope in arithmetic progression, as in 159753. [*J.R.M.* 4:170; July 1971.]

Integer, plateau. An integer with all like intermediate digits, which differ from its like end digits, as in 47774. [*J.R.M.* 4:169; July 1971.]

Integer, valley. An integer with digits that, reading from the left, steadily decrease to a minimum and then steadily increase to a final right-hand digit, as in 985234. [*J.R.M.* 4:170; July 1971.]

Integer, regular valley. A valley integer with digits on each slope in arithmetic progression, as in 864234. [*J.R.M.* 4:170; July 1971.]

Integer, undulating. An integer in which the alternate digits are consistently greater than or less than the digits adjacent to them, as in 415362. [*J.R.M.* 4:169; July 1971.]

Integer, smoothly undulating. An undulating integer in which only two distinct digits are present, as in 2525252. [*J.R.M.* 4:169; July 1971.]

Instant Insanity. Trade name of a popular puzzle consisting of four multicolored unit cubes, each of which has its faces painted red, blue, white, or green in a definite manner. The puzzle is to assemble the four cubes into a $1 \times 1 \times 4$ rectangular prism such that all four colors appear on each of the four long faces of the prism. Of 82,944 possible rectangular prisms that can be arranged from them, only two distinct arrangements satisfy the required conditions.

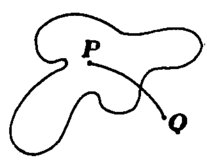
Irregular operation. See *Illegal operation.*

Jeu de Baguenaudier. See *Chinese rings.*

Jeu du Taquin. The Fifteen, or Boss, puzzle; also known as *Diablotin* in France, and sometimes called *Imp.*

Jordan curve. Any simple plane curve. See also *Jordan's theorem; simple curve.*

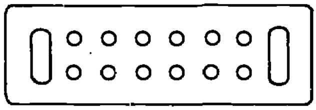
Jordan's theorem. In a plane, every closed curve that does not cross itself divides the plane into one interior region and one exterior region. Such a curve is called a simple curve. Any continuous line connecting a point in the interior with a point in the exterior (such as *PQ* in the figure shown) must intersect the curve. Every simple curve in the plane is topologically equivalent to a circle.



Jourdain's card paradox. A well-known logical paradox: One side of a card reads, "The sentence on the other side of this card is TRUE"; the other side of the card reads, "The sentence on the other side of this card is FALSE."

Kakeya needle problem. What is the plane figure of least area in which a line segment of unit length can be rotated 360°? [Sôichi Kakeya, Japan, 1917.]

Kalah. An African board game somewhat like Oware; in addition to the twelve compartments there is a larger oval compartment (*kalah*) at either end of the board. The purpose of the game is to accumulate as many playing pieces as possible in the large oval kalah at the right.



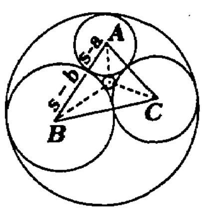
Kaprekar's constant. The unique self-reproducing integer 6,174, which terminates Kaprekar's routine when this routine is applied to any four-digit integer (not all four digits alike) in decimal notation. [*Scrip.M.* 15:244; 1949.]

Kaprekar's routine. Consists of rearranging the digits (not all alike) of an integer, N_0, to form the largest and smallest possible integers, finding their difference, N_1, and applying the ordering-subtraction operation to N_1 and to the subsequent differences until a self-producing integer or a regenerative loop is obtained. [*M.Mag.* 45:121; May 1972.]

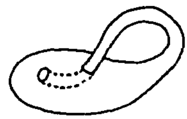
Kepler's star polyhedra. Three-dimensional star polyhedra formed by extending the edges of an icosahedron or those of a pentadodecahedron.

Kirkman's schoolgirls problem. As originally enunciated, to arrange fifteen things in different sets of triplets. [*Lady's and Gentleman's Diary*, 1850.]

Kiss Precise. The title of a poem by Frederick Soddy (1936) in which he gives, in verse and without symbols, the relationship between the radii of mutually tangent circles. The relationship in question is $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = \frac{1}{2}(\alpha + \beta + \gamma + \delta)^2$; where α, β, γ are the curvatures of three given circles A, B, and C expressed in terms of the sides a, b, c, of $\triangle\ ABC$, and δ is the curvature of the inner circle whose radius is r; thus $\delta = 1/r$, and $\alpha = 1/(s - a)$, $\beta = 1/(s - b)$, and $\gamma = 1/(s - c)$, where $2s = a + b + c$.



Klein bottle. A bottle-shaped, one-sided surface with no edges and no "inside" and no "outside"; it can be formed by inserting the small end of a tapering tube through one side of the tube and then spreading it so as to join the other end.



Knight's tour. A classic chessboard problem calling for a path formed by moving the knight in such a way that it will move successively onto every possible cell once and only once. If the knight can move from the last cell reached to the initial cell, then the tour is called a *re-entrant* path.

Knot. A simple closed curve in three-dimensional space. Both in a plane and in four-dimensional space all simple closed curves can be deformed, without crossing themselves, into circles; but in three-dimensional space some curves forming a knot cannot be so deformed.

Königsberg bridges. A classical problem. Is it possible to traverse a specific pattern of seven bridges in such a way as to cross each bridge once and only once? The problem was shown to be impossible by Euler, who, in generalizing it, laid the foundation of modern network (graph) theory.

Labyrinth. See *Maze*.

Latin square. A Latin square of the nth order is an array of n^2 cells, in n rows and n columns, in which n letters consisting of n a's, n b's, and so on, are arranged in the cells so that the n letters in each row and each column are different.

Lattice. A lattice is a set of points with integral coordinates, in a plane or in space, with respect to a Cartesian reference system. The points belonging to this set are called *lattice points*. Two fundamental properties of a plane lattice are these:

1. It is possible to draw through any given lattice point infinitely many lines that do not pass through any other lattice point.
2. Any straight line through two given lattice points must pass through an infinity of other lattice points.

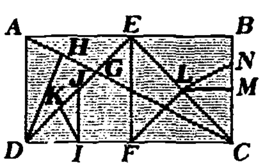
Lehmer's photoelectric number-sieve. An electronic device that factorizes large numbers with unbelievable rapidity.

Life. A sophisticated solitaire recreation invented by John H. Conway that simulates population changes by using "genetic laws" relating to births, deaths, and survivals; played on a large checkerboard or a Go-board with a large number of counters of two colors.

Linkage. A mathematical or drawing device consisting of a combination of bars or pieces pivoted together so as to turn about one another, usually in parallel planes; often used for transmitting motion.

Lissajous's figures. This refers to the family of curves that are described by a point whose motion is the resultant of two simple harmonic motions in perpendicular directions. Since the motions have different periods and different amplitudes, the variety of interesting patterns obtainable is seemingly endless. Lissajous's figures can be produced mechanically by suitably arranged swinging pendulums (harmonograph), by means of differential rotating gear wheels (spirograph), or by means of an electrical device (oscillogram).

Loculus of Archimedes. An ancient tangram puzzle consisting of fourteen tiles made by dissecting a rectangle as shown in the figure, the length of the original rectangle being twice its width; M, N, L, J, I, and H are midpoints, and $\overline{IK}$ extended passes through A.

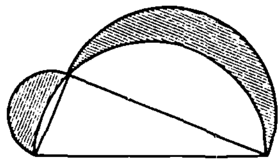


Logic machine. In general, an electronic computer designed or programmed to manipulate the symbols of thought; for example, to translate from one language to another, to play a game of checkers or Nim, to prove theorems in symbolic logic, and so on.

Logic problems. See *Inferential problems.*

Lucas numbers. The Lucas sequence $\{L_n\}$ is defined by $L_1 = 1$, $L_2 = 3$, $L_n = L_{n-1} + L_{n-2}$ for $n \geq 3$. Thus $\{L_n\} = \{1, 3, 4, 7, 11, 18, \cdots\}$.

Lunes of Hippocrates. The two lunes, or crescents, formed by semicircles having the legs and hypotenuse of a right triangle as diameters. The sum of the areas of the two lunes is equal to the area of the triangle.



McKay's theorem. Given $a/b < c/d$, where a, b, c, and d are integers, then $(a + b)/(b + d)$ is between a/b and c/d.

Magic circles. Circles so constructed that their points of intersection can be numbered in such a way that the sum of the integers lying on any given circle is equal to the sum of the integers lying on each of the other circles.

Magic constant. The constant sum of the numbers in each row, column, and main diagonal of a magic square. For a normal magic square the constant is given by $\frac{1}{2}n(n^2 + 1)$.

Magic cube. An $n \times n \times n$ magic cube consists of a series of integers so arranged that any column or row of n integers and the four main diagonals (each containing n integers) through the center of the cube add up to the same magic constant.

Magic hexagon. A unique configuration of nineteen hexagonal cells arranged contiguously to form an overall hexagonal shape. The cells are numbered with the integers 1–19 in such a way that the sum of the integers in any straight line of edge-joined cells is 38.

Magic square. A square array of n numbers such that the sum of the n numbers lying in any row, column, or main diagonal is some constant. The numbers may be positive or negative integers, or fractions. Generally, magic squares are formed from the first n natural numbers, in which case they are called normal magic squares.

Magic star. A five-pointed star (pentagram) with its five points of intersection and its five "points" so numbered that the sum of the magic constant for each straight line is 24. The numbers used are the integers 1 through 12, with the 7 and the 11 omitted.

Malfatti problem. To determine the sizes of three nonoverlapping circles of the greatest combined area which could be cut from a given triangle. [1803.]

The converse problem is to find the triangle of least area which can enclose three nonoverlapping circles of given radii. (The converse problem was presumably not considered by Malfatti.)

Map coloring. A generalization of the four-color map problem, which seeks to determine the smallest number of colors sufficient for coloring the countries of a given map. It is known that all possible maps with thirty-eight or fewer countries in the plane or on a sphere can be colored with just four colors. A general proof that four colors are sufficient for all possible maps in the plane has not as yet been found.

Marelle. See *Nine Men's Morris.*

Martingale. Specifically, a system of betting such that in a sequence of bets, losses are recovered by progressively increasing the stakes.

Mascheroni constructions. Problems of geometric construction calling for the use of the Euclidean compasses (collapsible) only.

Masculine numbers. See *Feminine numbers.*

Maxigon. The maximum-sided polygonal face possible in a member of a *V*-family of polyhedra (q.v.). Each member will have its own maxigon, varying in the number of sides, and the largest of the family will occur in that member in which $n - 1$ vertices lie in a plane and one outside it. The vertices of the plane define this maxigon. [John McClellan, *J.R.M.* 3:58–60; Jan. 1970.]

Maze. Specifically, a complex network of intercommunicating paths or passages; idealized, the paths are the branches of an Euler graph or network, and the places where two or more paths meet are *nodes.*

Mersenne numbers. Numbers of the form $2^n - 1$, where n is an integer.

Mill. See *Nine Men's Morris.*

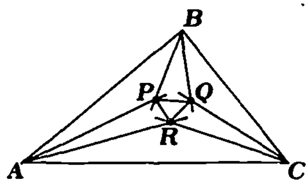
Misère. A term sometimes applied to games when the object is to try to *avoid* the customary win; for example, Nim, pebbles, losing checkers, or ticktacktoe (forcing the opponent to make a straight line).

Möbius band. If one edge of a long strip of paper with parallel edges is twisted through 180° and the ends of the strip are then fastened, the resulting surface is a Möbius band; it has only one "side" and only one bounding edge. Any point on the strip can be joined to any other point on the strip by a curve lying wholly on the strip and not crossing the bounding edge. See also *Paradromic rings.*

Moiré pattern. In general, a pattern produced whenever two periodic structures are overlapped. The interacting figures often consist of straight lines, curves, or dots, but not necessarily; the basic requirement is that the figures have some sort of solid regions and some open regions (cf. *Moiré silk*).

Monomino. A single-square polyomino.

Morley's theorem. If the trisectors of the angles of a triangle are drawn so that those adjacent to each side intersect, the intersections are the vertices of an equilateral triangle. [The theorem was enunciated by Frank Morley in 1899 and proved fifteen years later by W. E. Philip.]



Mosaic. A surface decoration made by inlaying in patterns small pieces of colored glass, stone, or wood, such as in tiled or parquet floors. Many mosaics are tessellations, although the basic requirement is that they cover the plane completely.

Multigrade. A multigrade is a particular relationship between sets of numbers and their powers. For example:

a) $1^2 + 6^2 + 8^2 = 2^2 + 4^2 + 9^2$
b) $1^3 + 6^3 + 7^3 + 17^3 + 18^3 + 23^3 = 2^3 + 3^3 + 11^3 + 13^3 + 21^3 + 22^3$

Multimagic squares. A magic square is multimagic of degree p if the square formed by replacing each element by its kth power is magic, for every k from 1 to p.

Multiperfect numbers. Any integer such that the sum of its divisors plus the given integer is equal to an integral multiple of the integer itself. For example, the sum of the factors of 120 is 240, and $240 + 120 = 360 = 3(120)$. In general, $s(N) + N = kN$, where $s(N)$ represents the sum of the divisors of N and k is an integer greater than 1.

Mutession. A tiling relationship in which the plane is covered by pairs of polygons such that each polygon in the pair tessellates the other in the sense that a finite number of congruent polygons that are similar to one of the two polygons can be arranged in a nonoverlapping way to form the other polygon. [R. W. Meyer, *M.Tchg.*, no. 56, pp. 24–27; Autumn 1971.]

Mutuabola. Name designating the graphs of $y^x = x^y$ and $x \log y = y \log x$. [R. R. Rowe, *J.R.M.* 3:176–78; July 1970.]

Mystic hexagram. Refers to the famous theorem of projective geometry, first enunciated by Pascal: If a hexagon be inscribed in a conic, then the points of intersection of the three pairs of opposite sides are collinear, and conversely.

Napoleon's problem. A classical Mascheroni construction problem: Given two diagonally opposite corners of a square, find the other two corners using only a compass.

Narcissistic number. Any number that can be represented in some way by a mathematical manipulation of the digits of the number itself. Many varieties are possible, including digital invariants and visible representations. Examples of narcissistic numbers would include the following:

$$371 = 3^3 + 7^3 + 1^3$$
$$407 = 4^3 + 0^3 + 7^3$$

Nasik magic square. Same as a diabolic, panmagic, or pandiagonal square.

Natural number. Any member of the infinite set $\{1, 2, 3, 4, \cdots\}$.

Necklace. A closed circular chain of beads, usually of two colors. [Dudeney; Gardner, *New Mathematical Pastimes from Scientific American*, p. 240.] Cf. also, *necklet*, as in Frederick Soddy's poem *The Hexlet*, which speaks of spheres as "a necklet of graded beads" (1936).

Net. A plane figure that can be "folded" into a three-dimensional model of a given polyhedron.

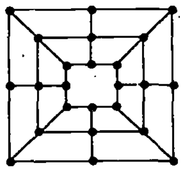
Network. See *Graph*.

Nim. In its most general form, a game for two players in which any number of counters are divided arbitrarily into several piles. Playing alternately, each player selects one of the piles and withdraws any number of counters, but at least one; the player drawing the last counter (or counters) wins. [C. L. Bouton, 1902.]

Nimatron. An electronic machine (computer) designed to play a perfect game of Nim by virtue of the fact that it operates on the binary system. [Edward U. Condon, 1940.]

Nimrod. An improved Nim-playing robot machine developed about 1951.

Nine Men's Morris. An old board game (also known as *triple hopscotch*, *Marelle*, and *Mill*) for two players, on a board as shown. Each player begins with nine counters; the object is to place three pieces in a row, which allows him to confiscate a piece from his opponent. The first player reduced to two pieces loses.



Normal magic square. Any magic square formed from the first n^2 natural number.

Normal number. Any number in whose decimal expansion all digits occur with equal frequency and all blocks of digits of the same length occur with equal frequency.

Normal piling. See *Spherical close-packing.*

Noughts and crosses. See *Ticktacktoe.*

Number base. A numeral used as the basis of any ciphered numeration system that uses the principle of position. If the base numeral is designated as b, then the system in base b will require the use of exactly b digits, viz., 0, 1, 2, 3, $\cdots$ $(b - 1)$. Any number N in this system may be expressed as the sum of successive powers of the base b: thus $N = a_0\,(b)^0 + a_1\,(b)^1 + a_2\,(b)^2 + a_3\,(b)^3 + \cdots$, where the coefficients $a_0, a_1, a_2, a_3, \cdots$ represent any of the b digits is used in the system.

Number giants. Although "large" is clearly a relative term, extraordinarily large numbers are encountered in the physical sciences and astronomy. Such numbers are sometimes referred to as number giants; for example, the *googolplex*, $10^{(10^{100})}$, or *Skewe's number*, $10^{10^{10^{34}}}$.

Number mysticism. A somewhat loose term embracing numerology, ancient and medieval beliefs in the esoteric qualities of numbers (e.g., Gematria), the humor of Martin Gardner's Dr. Matrix, and other number curiosities and coincidences.

Number pleasantries. A broad term often used to include number oddities, curiosities, patterns, and paradoxes; number tricks; digital variations and identities; multigrades; palindromes; illegal operations; and so on.

Numeral. Any symbol or group of symbols used to designate a number or a constitutent of a number; specifically, any one of the so-called Hindu-Arabic numeral symbols, that is, 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Numerology. The pseudoscientific study of numbers; a spurious art, akin to astrology, in which numbers and number relations can allegedly be used to foretell the future, influence events, explain coincidences, reveal character, and so forth.

Oblong numbers. See *Rectangular numbers.*

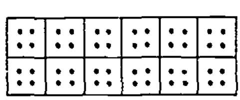
Octahedron. A convex octahedron is a solid figure bounded by eight plane polygons. Thus a hexagonal prism or the frustum of a hexagonal pyramid are octahedrons. A *regular* octahedron is bounded by eight congruent equilateral triangles; it has 6 vertices, 8 faces, and 12 edges.

Octonary numeration. A numeration system with base eight, or on the scale of eight; also known as *octic arithmetic.*

Optical illusion. As used in recreational mathematics, any diagram or drawing that creates a false mental image, invites a misinterpretation, or depicts an "impossible" physical reality.

Origami. Originally, the art of folding realistic animals, birds, fish, and other objects from a single sheet of paper without cutting, pasting, or adding decorations. In recent times these restrictions are sometimes overlooked. Paper folding of regular polygons and other geometric figures, although not unrelated, is strictly not a part of classical Origami.

Oware. A popular board game played by native children and adults mostly in West Africa. The game requires considerable skill and is played on a board divided into twelve compartments. Initially, each compartment contains four small objects. Sometimes twelve small holes are dug in the ground. The two players, using 48 pebbles, sit on opposite sides of the board or holes, using the stones as with Kalah.



Packing. A packing of convex bodies is an arrangement in which no two of the bodies have common interior points. Packing problems are encountered in crystallography, botany, virology, and other areas of physical science, as well as in the theory of numbers and information theory. See also *Close packing; Tessellations.*

Palindrome. Any number, symbol, word, or sentence that reads the same from left to right and vice versa. For example: 24942; · — — ·; 1,5,10,10,5,1; level; noon; "Madam, I'm Adam"; "Able was I ere I saw Elba."

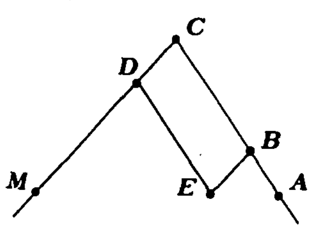
Palindromic number. An integer symmetrical to its middle. Otherwise, an integer that coincides with its reverse, as do 121 and 483384. Its coincidence ratio is 1.

Pandiagonal magic square. Any magic square in which all the broken diagonals as well as the main diagonals add up to the magic constant.

Pangram. An old form of playing with words in which the goal is to get the maximum number of different letters into the shortest possible sentence.

Panmagic square. Same as a pandiagonal or nasik square.

Pantograph. An instrument used for enlarging or reducing a given diagram, map, and so on. In essence, it is a linkage as here shown. Points M, E, and A are collinear; $BCDE$ is a parallelogram; and the ratio $AE/EM = AB/BC =$ constant. If any one of the points M, E, or A is kept fixed, the other two describe similar curves.

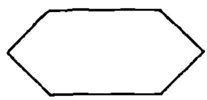


Paper folding. The practice of folding paper to form (1) specified geometric figures or (2) desired objects or toys. In the latter instance the art is called *Origami* (q.v.) and permits the use of scissors and other gadgets to assist and embellish forming the end product.

Paradox. A statement or situation that either is self-contradictory or contradicts a previously accepted axiom, theorem, or principle. In logic, *paradox* is generally regarded as synonymous with antinomy, although the term *paradox* is also correctly used in a broader sense to include intuitive contradictions such as optical illusions, "illegal" arithmetical operations, fallacious proofs in geometry or algebra, and unexpected properties of geometric figures.

Paradromic rings. A modification of the familiar Möbius band, given m half-twists (i.e., when one end of the original strip is turned through an angle of $m\pi$ radians). When m is even, a surface with two sides and two edges is obtained; when cut along the center line, this surface yields two rings each having m half-twists and linked together $m/2$ times. When m is odd, the surface obtained has only one side and one edge; when cut along its center this surface yields only one ring, which has $2m + 2$ half-twists (if $m > 1$, the ring is knotted).

Parhexagon. A parhexagon is a hexagon in which any side is both equal and parallel to the side opposite it.



Parity. Two integers are said to have the same parity if they are both even or both odd; if one is odd and the other is even, they are said to have different parity.

A "parity check" refers to a reasoning process that in some way depends on identification with odd and even numbers.

Parquet. Essentially the same as a tessellation; some parquets are not as "restricted" as a tessellation, but a parquet always covers the plane completely.

Partition (of an integer). The number of partitions $p(n)$ of an integer n is the number of ways n can be written as a sum of positive integers, $n = a_1 + a_2 + \cdots + a_k$, where k is a positive integer and $a_1 \geq a_2 \geq a_3 \geq \cdots \geq a_k$.

Pascal's triangle. A triangular array of numbers consisting of the coefficients of the expansion of $(a + b)^n$, for $n = 0, 1, 2, 3, \cdots$.

Path. A route in a graph that passes through no edge more than once.

Pathological curve. Any curve regarded as the limit of a sequence of polygons or a sequence of numbers; for example, the snowflake curve, the in-and-out curve, the crisscross curve, space-filling curves, and dragon curves.

Peasant multiplication. A method for multiplying any two integers by halving and doubling, rejecting remainders, as here shown.

39	53
19	106
9	212
4	424
2	848
1	1696

Only those numbers in the second column that correspond to odd numbers in the first column are added. Thus

$$39 \times 53 = 53 + 106 + 212 + 1696 = 2067.$$

The explanation of the algorithm rests on the properties of numbers expressed in the binary scale.

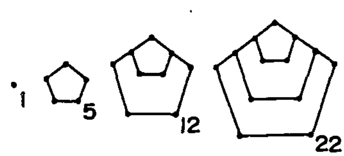
Pebbles. A game for two players. At the start of the game, an odd number of pebbles is placed in a pile. Taking turns, each player draws one, two, or three pebbles from this common pile. When all the pebbles have been drawn, the player who has an odd number of them in his possession is declared the winner.

Peg solitaire. A popular board game in England, the United States, and the USSR, where the board consists of thirty-three square cells arranged in the form of a Maltese cross. It is usually played with marbles. In France a 37-cell board is used, whereas in other countries both boards are in use.

Pendulum patterns. Intricate patterns traced by a ray of light through a tiny hole at the bottom of a compound pendulum as it swings over a sheet of photographic paper. Sometimes the oscillating pendulum is fitted with a pen. The traceries formed resemble Lissajous's figures.

Pentacubes. The 29 pieces formed by putting five congruent cubes together in all possible ways. [Theodore Katsanis, ca. 1960.]

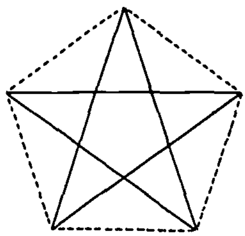
Pentagonal numbers. A class of polygonal numbers defined by the geometric figure they represent; thus for a pentagon:



They may also be defined as sums of special arithmetical sequences:

$$1 + 4 + 7 + 10 + \cdots + (3n - 2) = \frac{n(3n - 1)}{2}.$$

Pentagram. A regular star-shaped figure formed by joining each vertex of a regular pentagon with every other vertex and then deleting the sides of the pentagon. Regarded by the Pythagoreans as a symbol of their order, it was presumed to possess magic qualities.



Pentomino. A five-square polyomino.

Perfect digital invariant. Any integer that is equal to the sum of the nth power of its integers for some n; for example:

$$1634 = 1^4 + 6^4 + 3^4 + 4^4.$$

Perfect number. Any integer, the sum of whose divisors, excluding the given integer, is equal to the integer itself. The two smallest perfect numbers are $6 = 1 + 2 + 3$, and $28 = 1 + 2 + 4 + 7 + 14$.

In all, by 1971, twenty-three perfect numbers have been found, some with the aid of electronic computers. It has been shown that any number of the form $2^{n-1}(2^n - 1)$ is a perfect number, provided that $(2^n - 1)$ is a prime number.

Perfectly odd square. A perfectly odd square of order n is an $n \times n$ array of ones and zeros such that the sum along each row, each column, and all diagonals is odd. A 1×1 array consisting of the element 1 is a trivial example; perfectly odd squares of order 2, 3, and 4 do not exist.

1	1	0	0	1
0	1	1	0	1
0	1	1	0	1
1	0	0	1	1
1	0	1	0	1

Periodic decimal. See *Repeating decimal.*

Permutation. Any ordered arrangement that can be formed by using some or all of a finite set of entities.

Petersburg paradox. See *St. Petersburg paradox.*

Petrie polygon. A skew polygon or a zigzag in which the first and second edges are sides of one face of a regular polyhedron, the second and third edges are sides of another face of the given polyhedron, and so on around.

Phi. Known also as the golden mean, or golden number. It is usually denoted by the Greek letters phi (ϕ) or tau (τ), where $\phi = \frac{1}{2}(\sqrt{5} + 1) = 1.61803 \cdots$ and $1/\phi = \frac{1}{2}(\sqrt{5} - 1) = .61803 \cdots$. If a line is divided in accordance with the golden section, one part is ϕ times the other, or $1/\phi$ part of the whole.

Photoelectric factoring machine. Invented jointly by D. H. Lehmer and D. N. Lehmer (University of California, ca. 1933); a sophisticated mechanism, involving gears and photoelectric cells, that identifies the factors of huge numbers in a matter of minutes.

Phyllotaxis. A botanical phenomenon in the arrangement of leaves on a tree, the florets of a sunflower, the helical whorls of a fir cone or a pineapple, and so on, where the arrangements involve fractions whose numerators and denominators are Fibonacci numbers.

Pick's theorem. The area enclosed by a polygon on a lattice or geoboard equals one-half the number of border nails plus the number of interior nails minus 1.

Piling. See *Ball-piling.*

Plateau's problem. To find the surface of least area spanning a given contour. For practical purposes, any specific solution can be obtained, approximately, by fashioning a wire frame in the shape of the desired contour and dipping it in a solution of soap and glycerine.

Platonic solids. The five regular Platonic polyhedra include the *tetrahedron* (4 triangular faces), the *cube* (6 square faces), the *octahedron* (8 triangular faces), the *dodecahedron* (12 regular pentagons), and the *icosahedron* (20 triangular faces). These five only were known to the ancient Greeks.

In addition, four other regular polyhedra are now known—the so-called *stellated polyhedra,* two with star vertices and two with star faces. None of these four is a convex figure.

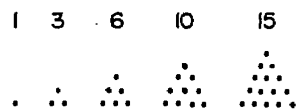
Polyaboloes. Flat shapes consisting of specified numbers of congruent isosceles right triangles joined edge to edge. They embrace diaboloes, triaboloes, tetraboloes, and so on. [*New Scientist* (England) 12:706; Dec. 1961.]

Polycube. A polyhedron formed by joining unit cubes.

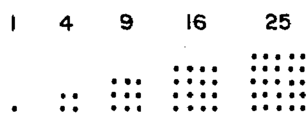
Polygonal knot. If a strip of paper is knotted once and carefully pressed flat, the folds will form a regular pentagon. All polygons with an odd number of sides may be produced in this manner.

Polygonal numbers. Polygonal numbers are a special kind of figurate numbers. They are characterized by their "shape."

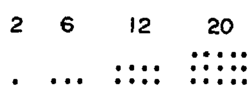
Triangular numbers:



Square numbers:



Also, *oblong numbers,* which are the sums of successive even numbers:



The theory is extended to include pentagonal, hexagonal, heptagonal, octagonal, etc., numbers. See also *Figurate numbers.*

Polyhexes. Flat shapes consisting of specified numbers of congruent hexagons joined edge to edge. They embrace trihexes, tetrahexes, pentahexes, and so on. [*Sci.Am.* 216:124; June 1967.]

Polyiamonds. Flat shapes consisting of specified numbers of congruent equilateral triangles joined edge to edge. They embrace diamonds, triamonds, tetramonds, and so on. [*New Scientist* (England) 12:316; Nov. 1961.]

Polyominoes. Flat shapes consisting of specified numbers of congruent squares joined edge to edge. Thus a domino consists of two congruent attached squares; a three-square figure is called a tromino, a four-square figure is a tetromino, a five-square figure is a pentomino, and so on. [Solomon Golomb, *A.M.M.* 61:672; Dec. 1954.]

Polytope. The bounded intersection of a finite number of closed half-spaces; or, technically, a subset of a Euclidean space that is the convex hull of a finite set of points. In a narrower sense, a regular figure in a space on n dimensions, $n > 3$.

Prime number. A positive integer greater than 1 that has no proper factors; or, more generally, an integer with absolute value greater than 1 that has no integral divisors or factors except itself and $\pm$ 1. Examples of prime numbers are 2, 5, 13, 79, 227. The number 1 is regarded as neither prime nor composite.

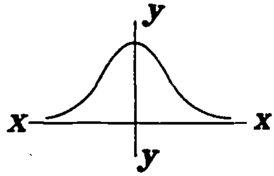
Primitive Pythagorean triple. If any two terms of a Pythagorean triple are relatively prime, then it is a primitive Pythagorean triple. For example: 3, 4, 5; 5, 12, 13; and 8, 15, 17.

Primitive semiperfect numbers. A number is primitive semiperfect if it is semiperfect but not divisible by any other semiperfect number. There are infinitely many primitive semiperfect numbers.

Prismoid. See *Antiprism.*

Probability board. See *Galton board.*

Probability curve. The well-known bell-shaped curve representing random-chance distribution or normal binomial frequency distribution; known also as the *Gaussian curve* or the curve of errors. One form of its equation is $y = ae^{-kx^2}$.



Proper divisors. The proper divisors of an integer comprise all its integral divisors, excluding the integer itself. Thus the proper divisors of 24 are {1, 2, 3, 4, 6, 8, 12}. They are also called *aliquot* divisors. (q.v.)

Pseudomath. A term coined by Augustus De Morgan to identify amateur or self-styled mathematicians, particularly circle-squarers, angle-trisectors, and cube-duplicators, although it can be extended to include those who deny the validity of non-Euclidean geometries. The typical pseudomath has but little mathematical training and insight, is not interested in the results of orthodox mathematics, has complete faith in his own capabilities, and resents the indifference of professional mathematicians.

Pyramidal numbers. "Solid" three-dimensional figurate numbers formed by summing polygonal-number series:

With a triangular base: 1, 4, 10, 20, 35, $\cdots$ $r(r+1)(r+2)/6$.
With a square base: 1, 5, 14, 30, 55, $\cdots$ $r(r+1)(2r+1)/6$.

Pyramidal numbers with triangular and square bases may represent pyramidal piles of symmetrical objects, say, spheres, where each layer represents a triangular or square number, respectively, and the pyramid is "uniformly solid." This is not true of pyramidal numbers with other bases, which will have one or more holes in the several layers of spheres.

Pythagorean triple. A set of three integers a, b, and c, that satisfies the identity $a^2 + b^2 = c^2$.

Quadratrix. A transcendental (i.e., nonalgebraic) curve, invented by Hippias (ca. 425 B.C.), which enables one to multisect an angle and to square the circle. One form of the Cartesian equation of this curve is $y = x \tan (\pi y/2)$.

Quadrature of the circle. One of the three famous problems of antiquity, it called for constructing a square equal in area to a given circle by means of Euclidean tools only, that is, straightedge and compasses. The solution is impossible under the specified conditions, as shown when Lindemann proved in 1882 that π is a nonalgebraic number.

Quadrille. A pattern of dominoes in which the numbers of dots are arranged in groups of four.

Queens problem. A chess problem that requires placing eight queens on a chessboard so that no one of them can take any other in a single move.

Quincunx. The Latin term for the familiar ⁙ pattern for "five" as it appears on dice and dominoes. Also used to designate the Galton probability board.

Rabbatment. Same as a *Schlegel diagram* (q.v.).

Radix. The base of any given numeration system or scale of notation.

Random clumping. Refers to questions such as, When objects are scattered at random, how many are hidden behind others? How many clumps of two or more will be formed?

Random digits. If, in a number of n digits, each of the ten digits 0, 1, 2, · · ·, 9 occurs approximately 10 percent of the time, the number is said to consist of random digits.

Recurring decimal. See *Repeating decimal.*

Recurring digital invariant. A sequence of sums of powers of a number ending with the original number. For example:

$$55:\ 5^3 + 5^3 = 250$$
$$250:\ 2^3 + 5^3 + 0^3 = 133$$
$$133:\ 1^3 + 3^3 + 3^3 = 55$$

Redundant number. Same as *Abundant number.*

Reflexible. A figure is reflexible if it is superposable with its image in a plane mirror. This is the ordinary meaning of the term *symmetrical.*

Repdigit. An integer composed of like digits, such as 77,777. [*J.R.M.* 5:123; Apr. 1972.]

Repeating decimal. A decimal in which all the digits (after a certain one) consist of a set of one or more digits repeated indefinitely; for example, .666 · · · or .01797979 · · ·. Every terminating decimal may be regarded as a repeating decimal; thus .25 = .25000 · · ·, and so on.

Repeating designs. Certain types of decorative design developed by repeating the same figure at regular intervals in the plane. They may or may not be tessellations and are often found in mosaics and parquets.

Rep-tile. A tile is a rep-tile of order n if exactly n copies of the tile may be used to form a pattern of the same shape as the tile.

Repunit. An integer consisting only of ones, such as 111 or 111,111. [*J.R.M.* 2:139; July 1969.]

Repunit prime. A prime number consisting only of ones, such as 11, or 1,111,111. Numbers consisting only of ones may be represented by the formula $\frac{10^n - 1}{9}$, where n gives the number of ones. To yield a repunit prime, a necessary—but not a sufficient—condition is that n be a prime.

Retrograde analysis. In chess problems, a technique of determining, from a given position, what has happened earlier in the game.

Reuleaux triangle. The simplest noncircular curve of constant width; it can rotate "snugly" within a square, maintaining contact continuously with all four sides of the square. [Franz Reuleaux, 1829–1905.]

Reversi. An old game played on a standard chessboard but in no way similar to checkers or chess. It is played with 64 counters having contrasting colors on their opposite sides, each player starting with 32 counters.

Rithomachy. Also known as *Rithmomachia*; a medieval number game, possibly of Greek origin. It was played on a double chessboard (8×16) and involved relations such as $42 = 36 + \frac{1}{6}$ of 36 and $81 = 72 + \frac{1}{8}$ of 72. Popular during the fourteenth and fifteenth centuries.

Rose curves. Curves whose polar equations are of the form $r = a \cos n\theta$ and $r = a \sin n\theta$, where n may be any positive real number.

Rotor. Any convex figure that can be rotated inside a polygon or polyhedron while constantly touching every side or face. [Michael Goldberg.]

Roulette. The curve generated by a fixed point on a curve as the curve rolls on another fixed curve (or straight line); for example, cycloids and trochoids.

Round robin. A tournament in which all the entrants play each other at least once, failure to win a contest not resulting in elimination from the tournament.

Rubber sheet geometry. A colloquial term that loosely describes topology from the layman's viewpoint.

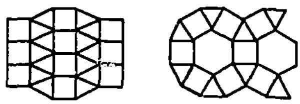
Russell's paradox. In a town that boasts of only one barber, all the men fall into one of two sets: those men who are shaved by the barber and those who shave themselves. To which set does the barber belong?

St. Petersburg paradox. A classic problem in probability theory. A penny is tossed until heads appears. If this occurs at the first throw, the bank pays the player £1; otherwise, the player throws again. If heads appears at the second throw, the bank pays £2; if at the third throw, £4; and so on, doubling every time. Thus, if the coin does not come down heads until the nth throw, the player then receives £2^{n-1}. What should the player pay the bank for the privilege of playing this game?

Scale of notation. See *Number base.*

Schläfli symbol (polyhedron). A polyhedron may be characterized by a Schläfli symbol $\{p, q\}$, which means that it has p-gonal faces, q at each vertex.

Schläfli symbol (tessellation). A symbolic representation specifying the nature of a semiregular tessellation by naming the polygons occurring at any vertex in the order in which they appear.



$[3^3, 4^2]$ $[3, 4, 6, 4]$

Schlegel diagram. A two-dimensional diagrammatic device that is intended to preserve the essential characteristics of a three-dimensional structure; also known as a *rabbatment.* Schlegel diagrams arise in connection with the study of polytopes and in the theory of graphs.

Sectio aurea. See *Golden section.*

Self-replicating digits. A set of n digits, no two alike, such that when they are arranged in descending order and reversed and the new number is subtracted from the original number, the same n digits reappear in the result. [Martin Gardner, *Sci.Am.*, Jan. 1965, p. 112.]

Semimagic square. A square that fails to be magic only because one or both of the main diagonal sums differs from the orthogonal sums.

Semiperfect numbers. A natural number n is called semiperfect if there is a collection of distinct proper divisors of n whose sum is n. In order that n be semiperfect, it is necessary, but not sufficient, that it be perfect or abundant.

A number is *primitive* semiperfect if it is semiperfect but not divisible by any other semiperfect number.

Semiregular solids. These are the thirteen *Archimedean solids* (q.v.).

Shoemaker's knife. See *Arbelos.*

Sickle of Archimedes. See *Arbelos.*

Sierpinski curve. A remarkable "pathological" curve that contains every interior point of a square and is nevertheless unicursal; its area is less than half that of the square.

Sieve of Eratosthenes. A procedure for identifying prime numbers, attributed to Eratosthenes (ca. 200 B.C.). Thus, from the set of natural numbers—

1. begin with 2, and delete all its multiples except itself;
2. find the next greater number not deleted, that is, 3, and delete all its multiples except itself;
3. find the next greater number not deleted, that is, 5, and delete all its multiples except itself;
4. continue in the same manner as far as desired.

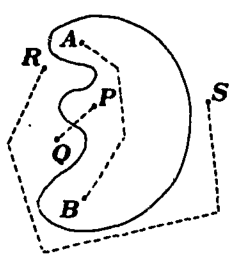
Those numbers not deleted are prime numbers.

As a practical method for identifying the primes, this procedure obviously has serious limitations.

Sim. A game for two people played on the six vertices of a regular hexagon. [Invented by Gustavus J. Simmons; see *Sci.Am.*, Jan. 1973, in Martin Gardner's column.]

Simple curve. A simple curve in a plane is a closed curve that does not cross itself; it has an interior and an exterior and thus separates the plane into two distinct regions. Hence—

1. as in the figure shown, any curve that contains both a point in the interior and a point in the exterior ($\overline{PQ}$) of a simple closed curve must of necessity intersect the given closed curve;
2. any two points in the interior (A,B) or in the exterior (R,S), may be joined by a broken-line curve that does not intersect the given closed curve.



Skeleton division. A long division in which most or all of the digits have been replaced by the same arbitrary symbol (such as × or *) to form a cryptarithm.

Skew polygon. A polygon whose vertices do not all lie in the same plane.

Snowball primes. A set of prime numbers whose digits follow a definite pattern. For example:

409; 4099; 40993; 409933; 4099339; 40993391;

and so on, which may or may not terminate.

Snowflake curve. Consider an equilateral triangle. Trisect each side and replace the center third of each by two sides of an equilateral triangle described on it outwards. Treat the resulting curve in the same way; continue this pattern indefinitely. The result is Von Koch's snowflake curve, which is infinitely long, has a finite area, and at no point possesses a tangent.

Sociable number. A number such that if after a certain number of steps in the process of successive additions of the divisors of the number, the original number is obtained. For example, Madachy gives the 19 divisors of 12,496 as 1, 2, 4, 8, 11, 16, 22, 44, 71, · · ·, 3,124, and 6,248; their sum is 14,288. There are 19 divisors of 14,288, and their sum is 15,472. Further, 15,472 has 9 divisors, whose sum is 14,536; this has 15 divisors, whose sum is 14,264; this has 7 divisors, whose sum is 12,496. [J. S. Madachy, *Mathematics on Vacation*, pp. 145–46; 1966.]

Soma cubes. The seven Soma pieces, created by Piet Hein, include all the different irregular, nonconvex polycubes that can be made by joining three or four unit cubes at their faces. The seven Soma pieces can be assembled into a solid cube, $3 \times 3 \times 3$.

Soma pieces. A subset of polycubes, namely, all the solid figures that can be formed by joining four unit cubes at their faces, yielding eight so-called tetracubes. Another subset consists of twenty-nine pentacubes.

Sophism. A fallacy in which faulty reasoning has been knowingly or deliberately injected. Zeon's so-called paradoxes are essentially mathematical sophisms.

Sorites. In logic, a form of argument involving several premises and one conclusion and admits of resolution into a chain of syllogisms, the conclusion of each of which is a premise of the next.

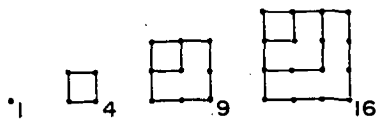
Spherical close-packing. A particular way of placing equal spheres in a box so that they are in horizontal layers and so arranged that each sphere is in contact with four spheres in the next lower layer, with four in the same layer, and with four in the next higher layer. Such an arrangement is also known as *normal piling;* it gives the greatest number of spheres with which the box can be filled.

Spirograph. A set of drawing instruments consisting essentially of a fixed plastic ring pinned to a drawing board and several smaller plastic disks whose teeth mesh with those of the fixed ring. A pencil inserted in a small hole in one of the small disks allows the disk to be rotated against the inner edge of the fixed ring and produces interesting curves and loops similar to Lissajous's figures.

Spirolateral. A geometric configuration derived from a logically constructed set of rules with the aid of conventional graph paper and appropriate rotations. [Frank C. Odds, *Math.Tchr.*, Feb. 1973, p. 121.]

Sprouts. A pencil and paper game, beginning with n spots on a sheet of paper. A move consists of drawing a line that joins one spot to another or to itself and then placing a new spot somewhere along the line. Lines may have any shape but must not cross lines or pass through previously made spots; no spot may have more than three lines emanating from it; the winner is the last person able to play. [J. H. Conway and M. S. Paterson, ca. 1966.]

Square numbers. A class of polygonal numbers that may be defined by the geometric figure they represent; thus for a square:



They may also be defined as sums of a special arithmetic sequence:

$$1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2.$$

It can be shown that every integer is the sum of at most four square numbers, not necessarily all different. See also *Gnomon; Polygonal numbers.*

Squared rectangle. Any rectangle that can be subdivided into unequal squares is known as a "perfect" rectangle. A squared rectangle is one that can be cut up into two or more squares, not necessarily unequal. "Squaring the square" means subdividing a given square into smaller squares, no two of which are alike.

Squaring the circle. See *Quadrature of the circle.*

Squaring the square. See *Squared rectangle.*

Stellated polyhedra. Polyhedra whose faces or vertex figures are "star polygons," that is, polygons with equal sides and angles, but not convex.

The term applies to some of the regular polyhedra as well as Archimedean polyhedra (with star-faces or star-vertices, or both).

Stomachion. See *Loculus of Archimedes.*

Street flexagon. A special type of flexagon whose faces, numbered 1, 2, 3, $\cdots$, n, may be made to appear in sequential order. [J. S. Madachy, *Mathematics on Vacation*, p. 73; 1966.]

String figure. A design or configuration made by taking a piece of flexible string from six to seven feet long, knotting the ends to form a closed loop, and then "weaving" or twisting this loop on the fingers to produce a desired configuration.

Strobogrammatic number. A number that is unchanged by plane rotation through 180°, such as 16891. [*M.Mag.* 34:182; Jan. 1961.]

Subfactorial. The subfactorial of an integer n is

$$n! \times \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots \frac{(-1)^n}{n!}\right].$$

For example, subfactorial 5 is $(120)\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120}\right) = 44.$

Superellipse. A curve having the same equation as an ellipse except that the exponent of the variables is $2\frac{1}{2}$ instead of 2. Created by Piet Hein. The three-dimensional form is known as a *superegg.*

Supertask. A problem situation or query leading to a paradox involving the concept of infinity or of infinite cardinals.

Symmetric. A figure is said to be symmetric if it admits of a certain number of symmetries (q.v.).

Symmetry. A symmetry, or a symmetry operation, is any combination of motions and reflections that leaves the figure unchanged as a whole. Any rotation or translation may be regarded as a combination of two reflections.

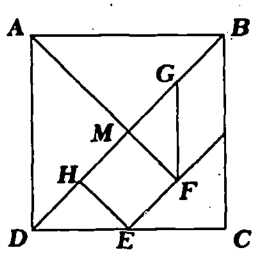
Tablut. An elaborate board game of Swedish origin; uses black and white pieces (king and warriors), all of which are moved like a rook in chess. [Martin Gardner, *Sci.Am.* 209:126; Oct. 1963.]

Tac Tix. A particular variation of Nim in which 16 counters are arranged in a 4×4 square. [Piet Hein, ca. 1950.]

Talisman hexagons, rectangles, and triangles. Number arrays similar to talisman squares.

Talisman square. An $n \times n$ array of the integers from 1 to n^2 such that the difference between any integer and its immediate neighbor (horizontally, vertically, or diagonally) is greater than some given constant.

Tangram. The Chinese tangram, an ancient geometric puzzle over four thousand years old, consists of seven pieces or tiles cut from a square as shown, where E, F, G, and H are, respectively, midpoints. The object of the tangram is to assemble the seven pieces to form common objects in silhouette. See also *Loculus of Archimedes.*



Tarry-Escott problem. The Tarry-Escott problem is that of finding two sets of integers, which may be assumed to be equal in number (since zero is allowed), such that those in each set have the same sum, the same sum of squares, and so on, up to and including the same sum of the kth powers.

Task problem. In chess, any problem that has maximum or minimum characteristics in relation to its space, medium, limitations, and thematic features. [T. R. Dawson, *Ultimate Themes*, 1938.]

Tau. An alternative symbol (τ) for the golden number

$$\phi = \frac{1}{2}(\sqrt{5} + 1) = 1.61803 \cdots$$

Tautochrone. The curve traced by a body moving without friction under the force of gravity such that the time required to reach a fixed point is the same regardless of the starting point.

Taxicab geometry. A variety of non-Euclidean geometry based on a lattice of points where the shortest "distance" between two points is not unique.

Teeko. A modification of ticktacktoe using a 5×5 board and four counters. [John Scarne, ca. 1950.]

Ternary numeration. A numeration system having base 3, or on the scale of 3, and therefore requiring only three digits: 0, 1, 2.

Base 10	*Base 3*
0	0
1	1
2	2
3	10
4	11
5	12
6	20
7	21
8	22
9	100
10	101
11	102
12	110
.	.
.	.
.	.

Tessellation. A plane tessellation is a collection of polygonal tiles that fit together with no overlapping or voids to cover the plane entirely. Or, it may be described as a two-dimensional honeycomb, that is, an infinite set of polygons fitting together to cover the entire plane exactly once, so that every side of each polygon belongs also to one other polygon; in short, a map with infinitely many faces.

Tessellation, regular. A tessellation consisting entirely of regular polygons, all exactly alike and meeting corner to corner, that is, no vertex of one polygon touches the side of another. There are exactly three possible regular tessellations.

Tessellation, semiregular. One in which two or more kinds of regular polygons are fitted together corner to corner in such a way that the same polygons, in the same cyclic order, surround every vertex. There are exactly eight semiregular tessellations.

Tesseract. See *Hypercube.*

Tetraflexagons. A group of four-sided paper structures similar to a hexaflexagon. The simplest tetraflexagon is a three-faced structure appropriately designated as a tri-tetraflexagon. There are at least six types of four-faced tetraflexagons, known as tetra-tetraflexagons. A hexa-tetra-flexagon has also been described.

Tetraflexatube. A flat, square-shaped flexagon that can be opened into a tube. By appropriate flexing along the boundaries of the right triangles, the tube can be turned completely inside out.

Tetrahedron. A convex tetrahedron is a solid figure bounded by four triangular faces; it is the simplest three-dimensional simplex. A regular tetrahedron is bounded by four congruent equilateral triangles; it has 4 faces, 4 vertices, and 6 edges.

Tetriamond. A polyiamond consisting of four triangles.

Tetromino. A four-square polyomino.

Ticktacktoe. A well-known game in which one player marks down only crosses and the other only ciphers, each alternating in filling in his mark in any one of nine cells in a square array. The player who first fills in three of his marks in a row, column, or diagonal is the winner. The game is simple; the strategy is not.

Tiling. See *Mosaic.*

Toetacktick. A modification of ticktacktoe in which the first player to get three in a row *loses.* [Mike Shodell.]

Tomahawk. A simple mechanical device by means of which an approximate trisection of an arbitrary angle can be effected. Although such a trisection is approximate, the tomahawk itself can be constructed with straightedge and compasses only.

Topology. A branch of geometry that deals with those properties of geometric figures that remain invariant under certain types of distortion or deformation; for example, a transformation that shrinks, twists, and so on, in any way without tearing.

Torus. Known also as an *anchor ring,* a circular torus is a doughnut-shaped three-dimensional "solid" figure produced by revolving a circle about an axis lying in its plane but not cutting the circle.

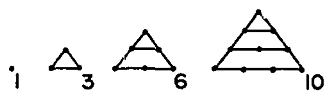
Tower of Hanoi. A classical puzzle presumably due to Lucas (1883) that consists of three pegs or spindles and eight circular disks of different diameters, each with a hole in the center. Initially, the eight disks are placed on one spindle so that the largest is on the bottom and the successive disks decrease in diameter with the smallest on top. It is required to shift the disks from one spindle to another in such a way that no disk shall ever rest on a disk smaller than itself and thus to transfer the original "tower" to another spindle with the disks finally arranged as they were on the initial tower.

Traveling salesman problem. If required to visit each of a given set of cities once, what route should a salesman take to make the total distance traveled a minimum?

Tree. A connected graph without circuits.

Triamond. A polyiamond consisting of three triangles.

Triangular numbers. A class of polygonal numbers that may be defined by the geometric figure they represent. Thus for a triangle:



They may also be defined as sums of a special arithmetic sequence:

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}.$$

It can be shown that every integer is either a triangular number or the sum of two (not necessarily different) or at most three triangular numbers. See also *Polygonal numbers.*

Trisection problem. Probably the best known of the three famous problems of antiquity—the others being the duplication of the cube and the squaring of the circle. An arbitrary angle cannot be trisected by using only the straightedge and compasses, although it can be done with the aid of conics, higher-plane curves, and transcendental curves.

Tromino. A three-square polyomino.

Twin primes. Successive primes with a difference of two, such as 17 and 19, or 821 and 823. They become relatively rare as primes get larger.

Unexpected egg paradox. A logical paradox similar to the paradox of the "unexpected hanging" (q.v.). [Michael Scrivin, *Mind,* vol. 60, July 1951; Martin Gardner, *The Unexpected Hanging and Other Mathematical Diversions,* pp. 11–23; 1963.]

Unexpected hanging paradox. A controversial logical paradox in which a judge sentences a prisoner on Saturday. "The hanging", says he, "will take place at noon on one of the seven days of next week. But you will not know which day it is until you are so informed on the morning of the day of the hanging." Assuming that the judge always kept his word, the prisoner's lawyer contended that the sentence could not possibly be carried out. Was he correct? [Ca. 1940; Michael Scrivin, *Mind*, vol. 60, July 1951.]

Unicursal curve. The path followed in tracing a given geometrical figure so that every line in it is traversed once and only once, although it is permitted to pass through any point of intersection (node) more than once.

Uniform polyhedrons. A polyhedron that has regular faces and that admits of symmetries which will transform a given vertex into every other vertex in turn. The Platonic polyhedra are uniform; so are the right regular prisms and antiprisms whose lateral faces are squares and equilateral triangles, respectively. There are exactly thirteen finite, convex uniform polyhedra; these are the Archimedean solids (q.v.).

Unique factorization theorem. Every integer greater than 1 can be represented in one and only one way as a product of prime numbers, disregarding the order of multiplication.

Unit fraction. Any fraction whose numerator is 1 and whose denominator is a positive integer $\neq 0$; for example,

$\frac{1}{2}, \frac{1}{5}, \frac{1}{32}$, and so on.

***V*-family.** A group of polyhedra having a like number of vertices. When these polyhedra differ as to the number of faces, F, each is called a *member* of the V-family. [John McClellan, *J.R.M.* 3:58–60; Jan. 1970.]

Venn diagrams. Diagrams employing overlapping and enclosing circles to show relationships between sets. [John Venn, 1834–1923.]

Versum. The sum of an integer and its reverse. Reiteration of the reversal-addition operation produces a *versum sequence*. [*M.Mag.* 45:186; Sept. 1972.]

Vertex. Either an endpoint of an edge or an isolated point of a graph.

Visible representation numbers. Any number that equals the sum of the squares (or cubes) of its digits taken in pairs or in halves, the sum of the factorials of its digits, and so on. For example:

$$1233 = 12^2 + 33^2$$
$$145 = 1! + 4! + 5!$$

Von Koch curve. See *Snowflake curve.*

Vux triangle. A triangle in which the measure of one of its three angles is one-half the measure of another of its angles. No vux triangle is equilateral; only two vux triangles are isosceles. [F. Cheney, *M.T.* 63:407; May 1970.]

Waring's problem. To show that for any integer n, there is an integer $K(n)$ such that any integer can be represented as the sum of not more than $K(n)$ numbers, each of which is an nth power of an integer. In particular, any integer can be represented as the sum of not more than four squares and as the sum of not more than nine cubes. The problem was solved in 1909.

Weird numbers. A weird number is an abundant number that is not semiperfect. There are infinitely many weird numbers; actually, the set of weird numbers has positive density. [S. Benkoski, *Am.M.Mo.* 79:774; Aug.–Sept. 1972.]

WFF'N PROOF. Trade name of a collection of twenty-one games of logic, ranging from very easy to rather challenging games. [Laymen E. Allen.]

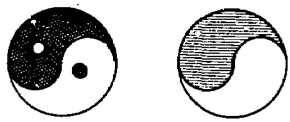
Whirling squares. A golden rectangle has the property that the removal of a square from one end of the rectangle leaves a similar rectangle, turned through 90°. If this process is continued indefinitely, a nest of squares is formed converging on a point P which is the pole of an equiangular (approximately) spiral passing through the points of division; hence, "whirling squares."

Wilson's theorem. This states that the number $(p - 1)! + 1$ is divisible by p if and only if p is a prime; for example, $6! + 1 = 721$ is divisible by 7, whereas $7! + 1 = 5{,}041$ is not divisible by 8.

Wythoff's game. A modification of Nim in which there are exactly two piles of counters; in each draw the player may select counters from either one or both piles, but in the latter event he must draw the same number from each pile. The player taking the last counter wins.

Yin and Yang. In Chinese religion and philosophy, Yin and Yang refer to two principles: Yin = dark, negative, and feminine; Yang = bright, positive, and masculine. The geometric pattern representing Yin and

Yang (shown below) has been used as a trademark as well as for decorative purposes.



Zeno's paradoxes. The arguments adduced by Zeno of Elea (ca. 450 B.C.) to prove that motion is impossible, regardless of whether distance or time is held to be infinitely divisible or to be made up of a large number of small, indivisible atomic parts. The four paradoxes include (1) the *Dichotomy*, (2) the *Achilles and the Tortoise*, (3) the *Arrow*, and (4) the *Stade*.

Zonahedra. Three-dimensional projections of n-dimensional hypercubes; their edges are all equal, and their faces are generally rhombs.